ABSTRACT

Universal algebra is an established field of mathematics and algebraic structures have multiple applications in mathematics itself, in engineering, and in computer science.

An algebra consists of a carrier set and a set of operations on this carrier set. The operations are deterministic, being functions from a number of arguments into the carrier.

There is a recent trend in computer science to study relaxed models of computation and implementations. Their results can be non-deterministic, or probabilistic, or weighted. One can say that such computational models compute with "effects". Effects have a neat categorical explanation via the notion of a monad.

Following this trend, and an initial investigation of two team members on probabilistic groupoids, we propose the study of probabilistic universal algebra. The operations in a probabilistic algebra are no longer deterministic — instead, the result of an operation is a probability distribution over the carrier set expressing the probability that an element is the result. Consider, for example, a two element set containing the elements 0 and 1. A probabilistic multiplication of 0 by 0 may return 0 with probability 0.9 and 1 with probability 0.1.

We will study probabilistic universal algebra by varying the complexity of the algebraic structures, by studying discrete and continuous probability distributions, and by moving from concrete universal algebra to categorical algebra. Our main goals in all cases are to provide notions of homomorphisms and congruences, equations and varieties, free algebras, Birkhoff-like theorems, as well as investigate particular concrete examples and possible applications.