Estimates for order of Nevanlinna matrices

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All solutions of an indeterminate Hamburger moment problem can be described with a Nevanlinna matrix. The four entries of this matrix are entire functions with the same exponential order. We are interested to determine this value.

We write the moment problem as a canonical system with Hamiltonian $H$. Here, $H : [0, L) \to \mathbb{R}^{2 \times 2}$ is a locally integrable function whose values are a.e. positive semi-definite. The corresponding canonical system is given by the equation

$$y'(x) = zJH(x)y(x), \quad x \in (0, L),$$

where $z \in \mathbb{C}$ and $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The so-called Hamburger Hamiltonians which appear here have a very particular form. Namely, $H$ has determinant zero, is piecewise constant, and constancy intervals accumulate only at $L$.

We obtain estimates for the order by transforming a given Hamburger Hamiltonian into (the Hamiltonian associated with) a Krein-string, and apply a theorem of I.S.Kac to evaluate the order of that string. Our result can be viewed as a generalisation of a theorem by Berezanskii in the 50s.

On the way, we leave the positive definite scheme and encounter Hamiltonians which take also negative definite matrices as values.