Reproducing kernel almost Pontryagin spaces

Harald Woracek

Institute for Analysis and Scientific Computing,
Vienna University of Technology, Wiedner Hauptstraße 8–10, Austria
http://www.asc.tuwien.ac.at/index.php?id=woracek, harald.woracek@tuwien.ac.at

An almost Pontryagin space $\mathcal{A}$ is an inner product space which admits a fundamental decomposition of the form $\mathcal{A} = \mathcal{A}_\geq \oplus \mathcal{A}_\leq$ with a Hilbert space $\mathcal{A}_\geq$ and a finite-dimensional negative semidefinite space $\mathcal{A}_\leq$. A reproducing kernel almost Pontryagin space is an almost Pontryagin space of complex-valued functions, with the property that all point evaluation functionals are continuous.

In this talk we address the following four items.

1° In the presence of degeneracy, it is not possible to reproduce function values as inner products with a kernel function in the usual way. We obtain a natural substitute for the reproducing kernel function, and study the relation between spaces and kernels.

2° Given an inner product space $\mathcal{L}$ of functions, does there exist a reproducing kernel almost Pontryagin space $\mathcal{A}$ which contains $\mathcal{L}$ isometrically? We characterise those spaces where the answer is “yes”. We show that, in case of existence, there is a unique such space $\mathcal{A}$ which contains $\mathcal{L}$ isometrically and densely (we speak of the reproducing kernel completion of $\mathcal{L}$).

3° A case of particular interest is that the space $\mathcal{L}$, though being not topologised, satisfies the algebraic axioms of a de Branges space. We show that in this case the reproducing kernel space completion (provided it exists) is a de Branges space. This topic is closely related to an indefinite variant of M.G.Krein’s theory of universal directing functionals.

4° The geometry of the reproducing kernel completion, in particular its degree of degeneracy, is an important invariant of $\mathcal{L}$. It plays a role in connection with Krein’s formula on the description of generalised resolvents and, thus, in several concrete problems related with the extension theory of symmetric operators. Prominent examples are, e.g., indefinite versions of the Hamburger power moment problem or the continuation problem for a positive definite function on an interval. We discuss one application in a little more detail.