

# A Study of Constrained Models for the Kinematic Analysis of the Human Knee Joint

I. Reichl\*<sup>†</sup>, W. Auzinger<sup>‡</sup>, H.B. Schmiedmayer<sup>††</sup> and E. Weinmüller<sup>‡</sup>

<sup>†</sup> University of Vienna, Institute of Sports Science, A-1150 Vienna, Austria; <sup>‡</sup> Vienna University of Technology, Institute for Analysis and Scientific Computing, A-1040 Vienna, Austria; <sup>††</sup> Vienna University of Technology, Institute of Mechanics and Mechatronics, A-1040 Vienna, Austria

*Keywords:* tibio-femoral joint; joint rotations; optimization; error compensation

## 1 Introduction

In a number of clinical applications such as ligament repair, joint replacement or prosthesis' design, the understanding of knee kinematics is of fundamental importance. Here, the determination of the joint rotation axes is crucial for the interpretation of joint kinematics. However, concurring definitions of knee joint axes based on anatomic landmarks are in use. Regarding any of these definitions, there is a large variability between observers and different sessions or trials. In order to reduce observer dependence, mathematical procedures based on movement analysis, so-called functional methods, have been developed. Presently, a number of alternative concepts exist allowing for the determination of joint rotation axes [1]. The finite helical axis (FHA) as an invariant description of joint displacements is a powerful tool; however, the FHA is not easy to interpret clinically [2]. Further approaches aim to bridge the gap between clinicians and engineers. The clinical protocol may be conserved while, subsequently, mathematical optimization is employed to reorient the clinically determined rotational axes, aiming at an optimal fit between the data and the underlying kinematic model [3,4]. Therein, the intact human tibio femoral joint (no ligament damage or osteoarthritis) is modeled as a compound hinge joint exhibiting only two rotational axes, flexion/extension (FE) and tibial rotation (TR).

It has been shown that, in an angular range of about 40° to 80° of knee flexion during weight bearing flexion exercises, the knee performs an almost plane movement. Furthermore, the flexion range from 0° to 40° is dominated by tibial rotation and flexion [3]. Optimization techniques determine the FE axis in the former angular regime, and the TR axis is found by minimizing the varus-valgus rotation in the latter regime. Here, it is usually assumed that the FE and TR axes are orthogonal and intersect. However, [5] state that the TR axis is anterior to the FE axis and not perpendicular to it.

A kinematic model that does not match the natural geometry of the joint is expected to yield unphysical displacements. It is the aim of this contribution to study one particular aspect in detail, namely the effect of an orthogonality constraint when applied to data from a knee exhibiting an arbitrary angle between the rotational axes.

## 2 Methods

To study the effect of different model constraints, a time series of kinematic data was generated via computer simulation of a compound hinge. The femur-fixed FE axis and the tibia-fixed TR axis intersect at an arbitrary angle. For the purpose of a first analysis of the proposed optimization models in the presence of noise, the angular relation of femur and tibia was distorted by Gaussian error.

The compound hinge optimization procedure is carried out after successful calculation of a FE axis within approximately 40° to 80° of flexion. Tibia movement is then described in a femur-fixed coordinate system with origin at the intersection of both axes and the  $z$ -axis coinciding with the FE axis. (The intersection can be derived as centre of a spherical joint.) In the initial configuration the relative orientation of the TR-axis with respect to the FE-axis is parameterized by an inclination angle  $\alpha$  and an azimuth  $\gamma$ . The FE angle  $\theta(t)$  and the TR angle  $\phi(t)$  describe the movement. In femur coordinates the orientation of the tibia reads

$$[E_{\text{tibia}}^{\text{model}}(t_n)] = R_z(\theta(t_n))R_z(\gamma)R_x(\alpha)R_z(\phi(t_n))R_x^T(\alpha)R_z^T(\gamma) \quad (1)$$

Here,  $R_x$ ,  $R_y$  and  $R_z$  are rotations about the principal axes. The objective function to be minimized,

$$\sum_{1 \leq n \leq N} \|[E_{\text{tibia}}^{\text{model}}(t_n)] - [E_{\text{tibia}}^{\text{data}}(t_n)]\|_F^2 \quad (2)$$

depends on the values  $\theta(t_n)$ ,  $\phi(t_n)$ ,  $\alpha$  and  $\gamma$ . For practical optimization, the unknown functions  $\theta(t)$ ,  $\phi(t)$  were replaced by fixed, reasonable ansatz functions [6], scaled by factors  $a_\theta$  and  $a_\phi$  which remain to be determined.

---

\*Corresponding author. Email: Irene.Reichl@univie.ac.at

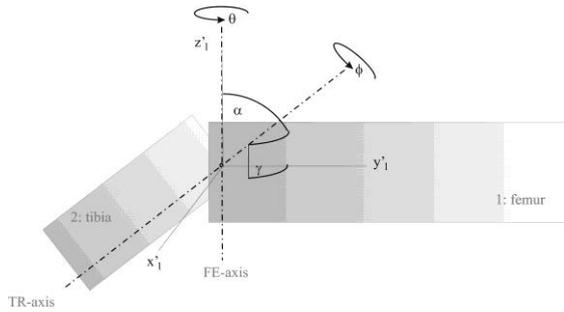


Figure 1. The inclination angle  $\alpha$  and the azimuth  $\gamma$  measure the relative orientation of the TR axis with respect to the FE axis. The joint rotation is given in terms of the FE angle  $\theta(t)$  and the TR angle  $\phi(t)$ .

It was the objective to quantify the effect of a model that does not match the original geometry. For this purpose two optimization models were compared: Model A imposing the orthogonality constraint on the objective function (1–2),  $\alpha=90^\circ$ , and model B with arbitrary  $\alpha$ . Thus, model A simultaneously optimized  $\gamma$ ,  $a_\theta$ , and  $a_\phi$  while model B optimized  $\gamma$ ,  $a_\theta$ ,  $a_\phi$ , and also  $\alpha$ .

### 3 Results and Discussion

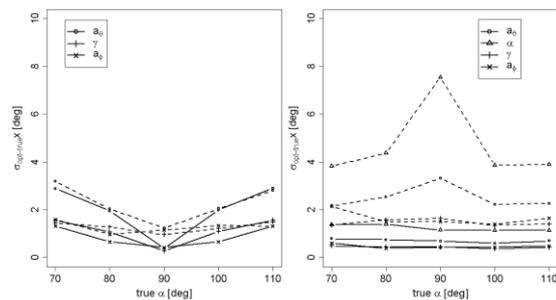


Figure 2. Model A (left) and B (right): difference of input parameters in the simulation, and optimized parameters. Gaussian noise of input data :  $\sigma=0.3^\circ$  (solid lines) and  $\sigma=3^\circ$  (dashed lines).

Angular noise of size  $0.3^\circ$  and  $3^\circ$  was introduced which corresponds to marker shifts of magnitude mm (cadaver measurements, medical imaging) or cm (skin-mounted markers), respectively.

The computations were performed using the MATLAB<sup>1</sup> Optimization Toolbox. Figure 2 shows the difference between the simulated input parameters and the optimized parameters for models A and B. In the input data the angle  $\alpha$  was varied between  $70^\circ$  and  $110^\circ$ . Therefore, the constrained model yields the most accurate results for  $\alpha=90^\circ$ . For small noise, the unconstrained model B may be preferred, for larger noise levels model A appears to be more convenient.

It turned out that the objective function does not show a pronounced minimum. The presence of

noise additionally deforms this function, which makes it more difficult to find its minimum. This should be attributed to an internal dependence between the angles, a subject currently under investigation.

### 4 Conclusions

Optimization models are built with the aim of error compensation with the risk of concealing realities not accounted for. This study could demonstrate that for errors in size typical for skin-fixed markers, it is justified to work with the orthogonality constraint previously applied by several independent research groups. However, soft-tissue motion induces static or dynamic shifts of markers versus underlying bones, requiring a velocity dependent model for noise compensation.

It is expected that for medical imaging data involving smaller noise it may be possible to optimize the kinematic data imposing a weaker constraint than the orthogonality constraint and, thus, permitting to reveal the natural tibio-femoral joint kinematics in more detail.

### References

- [1] Ehrig R.M., Taylor W.R., Duda G.N., and Heller M.O., A survey of formal methods for determining functional joint axes. *Journal of Biomechanics*, **40** (2007) 2150-2157.
- [2] Woltring H., de Lange A., Kauer J., and Huiskes R., Instantaneous helical axes estimation via natural, cross-validated splines, , in *Biomechanics: Basic and Applied Research Springer*, 121-128 (1987).
- [3] Marin F., Mannel H., Claes L., and Dürselen L., Correction of axis misalignment in the analysis of knee rotations, *Human Movement Science* **22** (2003) 285-296.
- [4] Baker R., Finney L., and Orr J., A new approach to determine the hip rotation profile from clinical gait analysis data, *Human Movement Science* **18** (1999) 655-667.
- [5] Hollister A.M., Jatana S., Sing A.K., Sullivan W.W., and Lupichuk A.G., The Axes of Rotation of the Knee. *Clinical Orthopaedics and Related Research*, **290** (1993) 259-268.
- [6] Moglo, K.E. and Shirazi-Adl, A., Cruciate coupling and screw-home mechanism in passive knee joint during extension-flexion, *J. Biomech.* **38** (2005) 1075-1083.

### Acknowledgments

This work was supported by the FWF (Austrian Science Fund), contract number T318-N14.

<sup>1</sup> MATLAB is a trademark of The MathWorks, Inc.

