# Operator Theory and Krein Spaces

Vienna University of Technology, December 19 - 22, 2019

# Program & Abstracts

# Thursday 19.12.2019

09'30 - 10'00 Registration (HS 5, 2nd floor)

- HS 5, 2nd floor-

| | |
|---|---|
| 10'00 - 10'30 | *Christiane Tretter:* Everything is possible for the domain intersection of an operator and its adjoint |
| 10'30 - 11'00 | *Roman Romanov:* Canonical systems in ideals of compact operators |

11'00 - 11'45 Coffee Break (Sem 03 A, 3rd floor) / Late Registration (HS 5, 2nd floor)

- HS 5, 2nd floor -

| | |
|---|---|
| 11'45 - 12'15 | *Jonathan Eckhardt:* Continued fraction expansions and generalized indefinite strings |
| 12'15 - 12'45 | *Vladimir Lotoreichik:* Optimization of lowest Robin eigenvalues on 2-manifolds and unbounded cones |

12'45 - 15'00 Lunch Break

| | - ZS 3, 7th floor - | - ZS 1, 8th floor - |
|---|---|---|
| 15'00 - 15'30 | *Igor Sheipak:* On embedding constants in Sobolev spaces. Application to spectral problems with coefficients-distributions. | *Seppo Hassi:* Stieltjes and inverse Stieltjes families of linear relations in Hilbert spaces and their representations |
| 15'30 - 16'00 | *Alexander K. Motovilov:* Equivalence between the complex-rotation and scattering-matrix resonances in the Friedrichs-Faddeev model | *Rudi Wietsma:* Factorizations, invariant subspaces and multi-valency |

16'00 - 16'45 Coffee Break (Sem 03 A, 3rd floor)

| | - ZS 3, 7th floor - | - ZS 1, 8th floor - |
|---|---|---|
| 16'45 - 17'15 | *Sergey Simonov:* Wave model of symmetric operators | *Lassi Lilleberg:* Minimal passive realizations of generalized Schur functions in Pontryagin spaces |
| 17'15 - 17'45 | *Vadim Mogilevskii:* Compressions of self-adjoint extensions of a symmetric operator in a Hilbert space | *Annemarie Luger:* Quasi-Herglotz functions |

# Friday 20.12.2019

Morning Session: Dedicated to the memory of Hagen Neidhardt.

- HS 5, 2nd floor -

| | |
|---|---|
| 09'00 - 09'45 | *Pavel Exner:* Spectral gaps of periodic quantum graphs |
| 09'45 - 10'30 | *Takashi Ichinose:* Magnetic Relativistic Schrödinger Operators and Kato's Inequality |
| 10'30 - 11'15 | *Vadym Adamyan:* Close singular perturbations of selfadjoint operators |

11'15 - 12'00 Coffee Break (Sem 03 A, 3rd floor)

- HS 5, 2nd floor -

| | |
|---|---|
| 12'00 - 12'45 | *Mark Malamud:* Scattering matrices, perturbation determinants, and trace formulas in the works of Hagen Neidhardt |
| 12'45 - 13'30 | *Valentin Zagrebnov:* The Howland-Evans-Neidhardt approach to approximation of propagators |

13'30 - 15'30 Lunch Break

| | - ZS 3, 7th floor - | - ZS 1, 8th floor - |
|---|---|---|
| 15'30 - 16'00 | *Artur Stephan:* On evolution semigroups and Trotter product operator-norm estimates | *Noema Nicolussi:* Self-adjoint extensions of infinite quantum graphs |
| 16'00 - 16'30 | *Anton Boitsev:* A model of several point-like windows in the resonator boundary with the Dirichlet boundary condition | *Jakob Reiffenstein:* Theorem of Hermite-Biehler for matrix-valued entire functions |

16'30 - 17'15 Coffee Break (Sem 03 A, 3rd floor)

| | - ZS 3, 7th floor - | - ZS 1, 8th floor - |
|---|---|---|
| 17'15 - 17'45 | *Nadezhda Rautian:* Semigroups for integro-differential equations with convolution memory terms | *Sergey Belyi:* Perturbations of L-systems |
| 17'45 - 18'15 | *Victor Vlasov:* Spectral analysis and representation of solutions of Volterra integro-differential equations with fractional exponential kernels | *Samuel Mohr:* Eigenvalues of Graphs |

15'30 Core Group Meeting COST Action CA18232 (Sem 03 C, 3rd floor)

19'00 Conference Dinner (Zwölf-Apostelkeller, Sonnenfelsgasse 3)

We walk from the university. For those who want to join: meeting point in front of HS 5, 2nd floor; departure 18'30

# Saturday 21.12.2019

Morning Session: Mathematical models for interacting dynamics on networks (COST Action CA18232)

- HS 5, 2nd floor -

| | |
|---|---|
| 10'00 - 10'30 | *Marjeta Kramar-Fijavz:* Bi-Continuous Operator Semigroups for Flows in Infinite Networks |
| 10'30 - 11'00 | *Pavel Kurasov:* Inverse Problem for Quantum Graphs: Complete Solution using Magnetic Control |

11'00 - 11'45 Coffee Break (Sem 03 A, 3rd floor)

- HS 5, 2nd floor -

| | |
|---|---|
| 11'45 - 12'15 | *Ivica Nakic:* Optimal control of parabolic equations using spectral calculus |
| 12'15 - 12'45 | *Petra Csomos:* Trotter-Kato Product Formula in Viewpoint of Numerical Analysis |

12'45 - 15'00 Lunch Break

| | - ZS 3, 7th floor - | - ZS 1, 8th floor - |
|---|---|---|
| 15'00 - 15'30 | *Petru Cojuhari:* On spectral analysis of Dirac operators | *Andrea Posilicano:* The semi-classical limit with delta potentials |
| 15'30 - 16'00 | *Alexander Makin:* On the basis property of root functions systems of Dirac operators with regular boundary conditions | *Luka Grubisic:* Convergence of contour integration methods for self adjoint operators |

16'00 - 16'45 Coffee Break (Sem 03 A, 3rd floor)

| | - ZS 3, 7th floor - | - ZS 1, 8th floor - |
|---|---|---|
| 16'45 - 17'15 | *Andrii Khrabustovskyi:* Geometric approximations of point interactions | *Jaroslav Dittrich:* Scattering along a curve in the plane |
| 17'15 - 17'45 | *Alexander Sakhnovich:* Discrete Dirac system and Arov–Krein entropy | *Nikos Yannakakis:* The angle along a curve and range-kernel complementarity |

# Sunday 22.12.2019

On Sunday the University building may be closed.
In this case, ring the bell at the main entrance and present your badge to the porter!

`- HS 5, 2nd floor -`

| | |
|---|---|
| 10'00 - 10'30 | *Rostyslav Hryniv:* Inverse scattering for reflectionless Schroedinger operators and generalized KdV solitons |
| 10'30 - 11'00 | *Matthias Langer:* Canonical systems whose Weyl coefficients have regularly varying asymptotics |

11'00 - 11'45 `Coffee Break (Sem 03 A, 3rd floor)`

`- HS 5, 2nd floor -`

| | |
|---|---|
| 11'45 - 12'15 | *Volodymyr Derkach:* De Branges-Pontryagin spaces and embedding of de Branges matrices with negative squares in generalized J-inner matrices |
| 12'15 - 12'45 | *Felix Schwenninger:* From input-to-state stability to semigroup perturbations |

`Closing`

# OTKR 2019 – Abstracts

## Close singular perturbations of selfadjoint operators

*Adamyan Vadym* Friday 10'30 (HS 5)

Let $H, H_1$ be unbounded selfadjoint operators in Hilbert space $\mathcal{H}$, $\mathcal{D}, \mathcal{D}_1$ and $R(z), R_1(z)$, $\mathrm{Im} z \neq 0$, are the domains and resolvents of $H, H_1$, respectively. $H_1$ is called close singular perturbation of $H$ if

$$1)\, \mathcal{D} \cap \mathcal{D}_1 \text{ is dense in } = \mathcal{H}; \quad 2)\, H_1 = H \text{ on } \mathcal{D} \cap \mathcal{D}_1;$$
$$3)\, R_1(z) - R(z),\ \mathrm{Im} z \neq 0, \text{ is a nuclear operator.}$$

The structures of the perturbation theory for close singular perturbations of selfadjoint operators are discussed with illustrations for the Schrdinger operators in $\mathbb{R}_3$ with interactions on low-dimensional manifolds.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Perturbations of L-systems

*Belyi Sergey* Friday 17'15 (ZS 1)

We study linear perturbations of Donoghue classes of scalar Herglotz-Nevanlinna functions and their representations as impedance of conservative L-systems. Explicit new formulas describing the von Neumann parameter of the main operator of a realizing L-system and the unimodular one corresponding to a self-adjoint extension of the symmetric part of the main operator are derived. This approach allows us to introduce a new concept of a *perturbed L-system*. In addition, we solve the inverse problem (with uniqueness condition) of recovering the perturbed L-system knowing the perturbation parameter $Q$ and the corresponding non-perturbed L-system. A concept of a unimodular transformation as well as conditions of transformability of one perturbed L-system into another one are discussed. Examples with differential operators that illustrate the obtained results are presented.

The talk is based on joint work with E. Tsekanovskiĭ.

**References**


[1] S. Belyi, E. Tsekanovskiĭ, *Perturbations of Donoghue classes and inverse problems for L-systems*, Complex Analysis and Operator Theory, vol. 13 (3), (2019), pp. 1227-1311.


– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## A model of several point-like windows in the resonator boundary with the Dirichlet boundary condition

*Boitsev Anton* Friday 16'00 (ZS 3)

A model of point-like windows in the resonator boundary for the case of the Dirichlet boundary condition is constructed. The model is based on the theory of self-adjoint extensions of symmetric operators in the Pontryagin space. We consider a situation when distance between windows tends to zero. A regularization is suggested.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## On spectral analysis of Dirac operators

*Cojuhari Petru* Saturday 15'00 (ZS 3)

Spectral properties, mainly important for scattering theory, of the Dirac operators describing particles in an external electromagnetic field will be discussed. Problems will treated for the general case, in an abstract framework, using direct methods of perturbation theory.

Results concerning the structure of the continuous spectrum, as well as estimates and asymptotic distribution of eigenvalues will be presented.

## Trotter–Kato Product Formula in Viewpoint of Numerical Analysis

*Csomos Petra* Saturday 12'15 (HS 5)

Trotter–Kato product formula provides the exponential of the sum of two unbounded operators as the limit of the product of the operators' scaled and squared exponentials. This formula might come in useful when proving the convergence of certain numerical methods, called operator splitting procedures, which approximate the solution of partial differential equations.

In the present talk we introduce the operator splitting procedures, show how their convergence can be proved by Trotter–Kato product formula, and present some ideas of generalisation from the literature. Especially, the results of Neidhardt and Zagrebnov (1999) will be treated, where instead of the exponential, certain functions were introduced in the product. We present such classes of numerical time discretisation methods which satisfy the author's assumptions on these functions. It is joint work with Eszter Sikolya (Budapest).

## De Branges-Pontryagin spaces and embedding of de Branges matrices with negative squares in generalized J-inner matrices

*Derkach Volodymyr* Sunday 11'45 (HS 5)

The notion of entire de Branges matrix $\mathfrak{E}(\lambda)$ with negative squares $\kappa$ is introduced. Associated to such matrix is a de Branges-Pontryagin spaces $\mathcal{B}(\mathfrak{E})$ with negative index $\kappa$. The problem of embedding of de Branges matrix $\mathfrak{E}(\lambda)$ with negative squares in generalized J-inner matrix $A(\lambda)$ is considered. This problem is proved to be solvable when the space $\mathcal{B}(\mathfrak{E})$ is invariant under the generalized backward shift operator. The theory of rigged de Branges-Pontryagin spaces is developed and then applied to obtain a solution of this embedding problem. A formula for factoring an arbitrary generalized J-inner entire matrix valued function into the product of a singular factor and a perfect one is found analogous to the known factorization formulas for J-inner matrix valued functions.

## Scattering along a curve in the plane

*Dittrich Jaroslav* Saturday 16'45 (ZS 1)

Quantum particles bounded to a curve in $\mathbb{R}^2$ by attractive contact $\delta$-interaction are considered. The curve is assumed $C^3$-smooth, non-intersecting, unbounded, asymptotically approaching two different half-lines (non-parallel or parallel but excluding the "U-case"). The interval between the energy of the transversal bound state and zero is shown to belong to the absolutely continuous spectrum, with possible embedded eigenvalues. The existence of the wave operators is proved for the mentioned energy interval using the Hamiltonians with the interaction supported by the asymptotic straight lines as the free ones.

## Continued fraction expansions and generalized indefinite strings

*Eckhardt Jonathan* `Thursday 11'45 (HS 5)`

Stieltjes continued fractions play a decisive role in the solution of the inverse spectral problem for Krein strings. Certain continued fractions of a modified form correspond in the same way to generalized indefinite strings. I will discuss under which conditions Herglotz–Nevanlinna functions allow such an expansion and use this to solve the inverse spectral problem for generalized indefinite strings with coefficients supported on a discrete set. These results are related to the Hamburger moment problem as well as multi-soliton solutions of particular integrable wave equations.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Spectral gaps of periodic quantum graphs

*Exner Pavel* `Friday 9'00 (HS 5)`

The last paper I coauthored with Hagen concerned quantum graphs. I am not going to discuss it here, instead I present some new results from the same area. The topic of the talk are gaps in the spectra of Schrödinger operators supported by metric graphs with a periodic structure, in particular, their dependence of the geometry and topology of the graph and on the vertex coupling. Two main questions will be addressed. The first is related to well known Bethe-Sommerfeld conjecture: we show that while generically the number of open gaps is infinite, there are situations where the spectrum contains a finite and nonzero number of gaps. In the second part, being motivated by a recent attempt to model the anomalous Hall effect, we discuss a vertex coupling noninvariant with respect to the time reversal. We show that, in contrast to more common examples of vertex coupling, the high-energy behaviour of the spectrum is then determined by the parity of the graphs vertices, and put this example into the context of some recent quantum graph results.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Contour integration methods for self adjoint operators

*Grubisic Luka* `Saturday 15'30 (ZS 1)`

Filtered subspace iteration coupled with Rayleigh-Ritz eigenvalue extraction is a recently revieved method in the form of the FEAST algorithm. The core of the algorithm is a numerical resolvent calculus based on the contour quadratures coupled with a perturbation analysis of the resolvent evaluation motivated by the results in Numerical Linear Algebra. We prove convergence rate which depends on the properties of the filter, even in the presence of (singular) perturbations. Perturbations which we consider can originate both from the projection/(domain truncation) of infinite dimensional operators (necessary to evaluate resolvents) or from the uncertainty in the parameters of the underlying problem.

The talk is based on joint work with J. Ovall, B. Parker and Jay Gopalakrishnan.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Stieltjes and inverse Stieltjes families of linear relations in Hilbert spaces and their representations

*Hassi Seppo* `Thursday 15'00 (ZS 1)`

Some analytic and geometric properties of Stieltjes and inverse Stieltjes families defined on a separable Hilbert space will be studied, including various minimal representations obtained by means of compressed resolvents of various types of linear relations. Also attention is paid to some peculiar properties of Stieltjes and inverse Stieltjes families. For instance, an analog for the notion of inner functions is introduced and characterized in an explicit manner. Also some transformers that naturally appear in the Stieltjes and inverse Stieltjes classes are studied and fixed points of these transformers are identified. These notions and results are closely connected to somewhat similar properties of a specific subclass of Schur functions.

The talk is based on some joint work with Yury Arlinskiĭ.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Inverse scattering for reflectionless Schrödinger operators and generalized KdV solitons

*Hryniv Rostyslav* `Sunday 10'00 (HS 5)`

In this talk, we discuss Schrödinger operators on the line with real-valued integrable reflectionless potentials $q$,

$$S_q := -\frac{d^2}{dx^2} + q.$$

In particular, we give a complete characterization of such operators in terms of their scattering data, sequences of eigenvalues and norming constants, and suggest an explicit formula producing all such potentials, thus completely solving the related direct and inverse scattering problems. Using the inverse scattering transform approach [1], we then describe all solutions of the Korteweg–de Vries (KdV) equation whose initial profile is an integrable reflectionless potential. Such solutions stay integrable and reflectionless for all $t \geq 0$ and can be called *generalized soliton solutions* of the KdV.

This research extends and specifies in several ways the previous work on reflectionless potentials [3, 4] and generalized soliton solutions of the KdV equation [2, 4]. The talk is based on a joint project with Ya. Mykytyuk (Lviv Franko National University, Ukraine).

### References


[1] C.S. Gardner, J.M. Green, M.D. Kruskal, and R.M. Miura, *Method for solving the Korteweg-de Vries equation*, Phys. Rev. Lett. **19**(19) (1967), 1095–1097

[2] F. Gesztesy, W. Karwowski, and Z. Zhao, *Limits of soliton solutions*, Duke Math. J. **68**(1) (1992), 101–150

[3] I. Hur, M. McBride, and C. Remling, *The Marchenko representation of reflectionless Jacobi and Schrdinger operators*, Trans. Amer. Math. Soc. **368**(2) (2016), 1251–1270

[4] V. A. Marchenko, *The Cauchy problem for the KdV equation with nondecreasing initial data*, in *What is integrability?*, Springer Ser. Nonlinear Dynam., Springer, Berlin, 1991, pp. 273–318


---

## Magnetic Relativistic Schrödinger Operators and Kato's Inequality

*Ichinose Takashi* `Friday 9'45 (HS 5)`

Corresponding to the classical relativistic Hamitonian symbol $\sqrt{(\xi - A(x))^2 + m^2}$ with *magnetic* vector potential $A(x)$, there are in the literature three kinds of relativistic Schrödinger operators on $L^2(\mathbb{R}^d)$, depending on *how to quantize this symbol*. One, $H_{A,m}$, is defined as the operator-theoretical *square root* of the nonnegative selfadjoint operator, the magnetic nonrelativistic Schrödinger operator $(-i\nabla + A(x))^2 + m^2$:

$$H_{A,m} := \sqrt{(-i\nabla + A(x))^2 + m^2}. \qquad (0)$$

The other two are pseudo-differential operators defined by oscillatory integrals as (with $f \in C_0^\infty(\mathbb{R}^d)$)

$$(H^{(1)}_{A,m} f)(x) := \tfrac{1}{(2\pi)^d} \iint_{\mathbb{R}^d \times \mathbb{R}^d} e^{i(x-y)\cdot(\xi + A(\frac{x+y}{2}))} \sqrt{\xi^2 + m^2} f(y) dy d\xi\,, \qquad (1)$$

$$(H^{(2)}_{A,m} f)(x) := \tfrac{1}{(2\pi)^d} \iint_{\mathbb{R}^d \times \mathbb{R}^d} e^{i(x-y)\cdot(\xi + \int_0^1 A((1-\theta)x + \theta y) d\theta)} \sqrt{\xi^2 + m^2} f(y) dy d\xi. \qquad (2)$$

(1) is through *Weyl quantization* with mid-point prescription and (2) a modification of (1) by Iftimie, Măntoiu and Purice. All these three operators differ in general, though they coincide for uniform magnetic field, and in particular for $A \equiv 0$, $H_{0,m} = H^{(1)}_{0,m} = H^{(2)}_{0,m} = \sqrt{-\Delta + m^2}$.

In this talk, we would like to handle *Kato's inequality in distributional form* for these relativistic Schrödinger operators, however here mainly for $H_{A,m}$ in (0), from joint work [HIL17] with Hiroshima and Lőrinczi.

**Theorem** (Kato's inequality). *Let* $m \geq 0$ *and* $A \in [L^2_{\mathrm{loc}}(\mathbb{R}^d)]^d$. *If* $u \in L^2(\mathbb{R}^d)$ *with* $H_{A,m}u \in L^1_{\mathrm{loc}}(\mathbb{R}^d)$, *then the distributional inequality holds:*

$$\mathrm{Re}[(\mathrm{sgn}\, u)H_{A,m}u] \geq H_{0,m}|u|, \text{ or } \mathrm{Re}[(\mathrm{sgn}\, u)[H_{A,m} - m]u] \geq [H_{0,m} - m]|u|. \tag{3}$$

*Here* sgn *is a bounded function in* $\mathbb{R}^d$*:* $(\mathrm{sgn}\, u)(x) = \begin{cases} \overline{u(x)}/|u(x)|, & \text{if } u(x) \neq 0, \\ 0, & \text{if } u(x) = 0. \end{cases}$

For the other two $H^{(1)}_{A,m}$ in (1) and $H^{(2)}_{A,m}$ and (2), there exist also Kato's inequalities [I 13].

**References**

[HIL17] F. Hiroshima, T. Ichinose and J. Lőrinczi: Kato's Inequality for Magnetic Relativistic Schrödinger Operators, *Publ. RIMS Kyoto University* **53**, 79–117 (2017).

[I 13] T. Ichinose: Magnetic relativistic Schrödinger operators and imaginary-time path integrals, *Mathematical Physics, Spectral Theory and Stochastic Analysis*, Operator Theory: Advances and Applications 232, pp. 247–297, Springer/Birkhäuser 2013.

— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —

## Geometric approximations of point interactions

*Khrabustovskyi Andrii* Saturday 16'45 (ZS 3)

In this talk we address the problem of an approximation of some singular Schrödinger operators describing the motion of a particle in a potential being supported at a discrete set. These operators are known as solvable models in quantum mechanics; the word *solvable* reflects the fact that their mathematical and physical quantities (spectrum, eigenfunctions, etc.) can be determined explicitly. Such models are also called *point interactions*.

One of the main problems arising in the theory of solvable models is their approximations by more "realistic" ones. In the talk we address the question of approximation of the so-called $\delta$ and $\delta'$-interactions using geometrical tools, namely, the Neumann Laplacians on thin domains with waveguide geometry. For the underlying operators we establish (a kind of) norm resolvent convergence and the Hausdorff convergence of their spectra. To approximated $\delta$-interactions we use waveguides with attached "room-and-passage" bumps, while for $\delta'$-interactions we utilize waveguides consisting of two thin straight tubular domains connected through a tiny window.

The talk is based on joint works with O. Post (in preparation) and G. Cardone *[J. Math. Anal. Appl 473(2) (2019), 1320-1342]*.

— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —

## Bi-Continuous Operator Semigroups for Flows in Infinite Networks

*Kramar-Fijavz Marjeta* Saturday 10'00 (HS 5)

We study transport processes on infinite (locally finite) metric graphs. On each edge $e_j$ of the network we take an evolution equation of the form

$$\frac{\partial}{\partial t} u_j(t,x) = c_j \frac{\partial}{\partial x} u_j(t,x)$$

and interlink them in the common nodes via some prescribed transmission conditions. We apply the theory of bi-continuous operator semigroups to obtain well-posedness of the problem in the $\mathrm{L}^\infty$-setting.

This is a joint work with Christian Budde.

— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —

## Inverse Problem for Quantum Graphs: Complete Solution using Magnetic Control

*Kurasov Pavel* `Saturday 10'30 (HS 5)`

Complete solution to the inverse spectral problem for the Schrödinger operator on a finite compact metric graph is presented. To solve the problem we use magnetic control allowing to collect spectral data without destroying the graph. Our approach is based on generalising ideas of the Boundary Control method for the Schrödinger equation in dimension one.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Canonical systems whose Weyl coefficients have regularly varying asymptotics

*Langer Matthias* `Sunday 10'30 (HS 5)`

For a two-dimensional canonical system $y'(t) = zJH(t)y(t)$ on the half-line $(0,\infty)$ whose Hamiltonian $H$ is positive semi-definite a.e., let $q_H$ be its Weyl coefficient. De Branges' inverse spectral theorem states that the assignment $H \mapsto q_H$ is a bijection from trace-normed Hamiltonians onto the set of Nevanlinna functions. In this talk I shall answer the question when $q_H(ir) \sim i\omega a(r)$ as $r \to \infty$ where $\omega \in \mathbb{C} \setminus \{0\}$ and $a$ is a regularly varying function, i.e. $\exists\, \alpha \in \mathbb{R}$ such that $\lim_{r\to\infty} \frac{a(\lambda r)}{a(r)} = \lambda^\alpha$ for all $\lambda > 0$. Note that the class of regularly varying functions includes, e.g. $a(r) = r^\alpha (\log r)^{\beta_1} (\log\log r)^{\beta_2}$ with $\alpha, \beta_1, \beta_2 \in \mathbb{R}$ but also some oscillating functions. I shall also discuss the relation between $\omega$ and $a$ on one hand and properties of $H$ on the other hand. The talk is based on joint work with Raphael Pruckner and Harald Woracek.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Minimal passive realizations of generalized Schur functions in Pontryagin spaces

*Lilleberg Lassi* `Thursday 16'45 (ZS 1)`

Passive discrete-time systems are investigated in setting where incoming, outgoing and state spaces are Pontryagin spaces . In this case the transfer functions of passive systems, or characteristic functions of contractive operator colligations, are generalized Schur functions. The existence of optimal and $^*$-optimal minimal realizations for generalized Schur functions are proved. By using those realizations, a new definition, which covers the case of generalized Schur functions, is given for defects functions. A criterion due to D.Z. Arov and M.A. Nudelman, when all minimal passive realizations of the same Schur function are unitarily similar, is generalized to the class of generalized Schur functions. The approach used here is new; it relies completely on the theory of passive systems.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Optimization of lowest Robin eigenvalues on 2-manifolds and unbounded cones

*Lotoreichik Vladimir* `Thursday 12'15 (HS 5)`

In this talk, we will focus on optimization for the lowest eigenvalue of the Robin Laplacian with a negative boundary parameter on a compact, smooth, simply-connected, two-dimensional manifold with $C^2$-boundary of a fixed length. The main novelty compared to the better-understood Euclidean case is that the eigenvalue is optimized in the sub-class of manifolds, for which the Gauss curvature satisfies the pointwise inequality $K \le K_\circ$ for a fixed constant $K_\circ \ge 0$. This constraint on the curvature naturally enters into the problem. Our main result can be concisely formulated as follows: *the geodesic disk on the manifold of the constant Gauss curvature* $K_\circ$ *is a maximizer.*

Moreover, we will discuss a result on the optimization of the lowest Robin eigenvalue on an unbounded three-dimensional Euclidean cone $\Lambda$ with a $C^2$-smooth, simply-connected cross-section $\Lambda \cap \mathbb{S}^2$ of a fixed perimeter. We prove that the cone with a circular cross-section is a maximizer.

This talk is based on a joint work with Magda Khalile.

-----------------------------------

## Quasi-Herglotz functions

*Luger Annemarie* `Thursday 17'15 (ZS 1)`

A quasi-Herglotz function is by definition a linear combination of Herglotz (Nevanlinna) functions. In this talk we are going to give different characterizations for these functions and discuss some important subclasses. Finally we relate to other areas.

This is joint work with Mitja Nedic.

-----------------------------------

## On the basis property of root function systems of Dirac operators with regular boundary conditions

*Makin Alexander* `Saturday 15'30 (ZS 3)`

In the present paper, we study the Dirac system

$$B\mathbf{y}' + V\mathbf{y} = \lambda\mathbf{y}, \tag{1}$$

where $\mathbf{y} = col(y_1(x), y_2(x))$,

$$B = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad V = \begin{pmatrix} 0 & P(x) \\ Q(x) & 0 \end{pmatrix},$$

the functions $P(x), Q(x) \in L_1(0, \pi)$, with two-point boundary conditions

$$U(\mathbf{y}) = C\mathbf{y}(0) + D\mathbf{y}(\pi) = 0, \tag{2}$$

where

$$C = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad D = \begin{pmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{pmatrix},$$

the coefficients $a_{ij}$ are arbitrary complex numbers, and rows of the matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix}$$

are linearly independent. Spectral problems for operator (1), (2) with regular but not strongly regular boundary conditions are considered. The purpose of this paper is to find conditions under which the root function system forms a usual Riesz basis rather than a Riesz basis with parentheses.

-----------------------------------

## Scattering matrices, perturbation determinants, and trace formulas in the works of Hagen Neidhardt

*Malamud Mark* `Friday 12'00 (HS 5)`

The aim of my talk is to overview a part of our joint work with Hagen devoted to perturbation determinants and trace formulas for a pair of operators with the trace class resolvent difference. This work was initiated more than twenty five years ago.

The talk will be devoted to perturbation determinants and trace formulas for a pair of operators with the trace class resolvent difference. A formula for the scattering matrix of a pair of selfadjoint operators will be discussed too. Both topics are treated in the framework of boundary triplet approach to the extension theory of symmetric operators. More precisely, the scattering matrix and perturbation determinants are expressed by means of the Weyl function and boundary operators. Applications to boundary value problems for ordinary differential operators and elliptic operators in bounded or exterior domains will also be discussed.

The talk is based on our joint papers [1] – [8] partially written in collaboration with J. Behrndt and V. Peller.

### References


[1] Behrndt, J., Malamud, M.M., Neidhardt, H.: Scattering matrices and Weyl functions. Proc. London Math. Society **97**(3), (2008), 568–598.

[2] Malamud, M.M., Neidhardt, H.: On the unitary equivalence of absolutely continuous parts of self-adjoint extensions. J. Funct. Anal. **260**(3), (2011), 613–638.

[3] Malamud, M.M., Neidhardt, H.: Sturm-Liouville boundary value problems with operator potentials and unitary equivalence. J. Differential Equations **252**, (2012), 5875–5922.

[4] M.M. Malamud, H. Neidhardt, Perturbation determinants for singular perturbations, Russ. J. Math. Phys. **21**, (2014), 55–98.

[5] M. Malamud, H. Neidhardt, Trace formulas for additive and non-additive perturbations, Adv. Math. **274**, (2015), 736–832.

[6] Behrndt, J., Malamud, M.M., Neidhardt, H., Scattering matrices and Dirichlet-to-Neumann maps. J. Funct. Anal. **273**, (2017), 1970–2025.

[7] M. Malamud, H. Neidhardt, V.V. Peller, Analytic operator Lipschitz functions in the disk and a trace formula for functions of contractions, Funct. Anal. Appl. **51** (3), (2016), 185–203.

[8] M.M. Malamud, H. Neidhardt, V.V. Peller, Absolute continuity of spectral shift, J. Funct. Anal. **276**, (2019), 1575–1621.


– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Compressions of self-adjoint extensions of a symmetric operator in a Hilbert space

*Mogilevskii Vadim* `Thursday 17’15 (ZS 3)`

Let $A$ be a symmetric possibly nondensely defined operator in the Hilbert space $\mathfrak{H}$ with equal deficiency indices $n_\pm(A) \le \infty$. A self-adjoint linear relation $\widetilde{A} \supset A$ in some Hilbert space $\widetilde{\mathfrak{H}} \supset \mathfrak{H}$ is called an exit space extension of $A$; such an extension is called finite-codimensional if $\dim(\widetilde{\mathfrak{H}} \ominus \mathfrak{H}) < \infty$. We study the compressions $C(\widetilde{A}) = P_{\mathfrak{H}}\widetilde{A} \upharpoonright \mathfrak{H}$ of exit space extensions $\widetilde{A} = \widetilde{A}^*$. For a certain class of extensions $\widetilde{A}$ we parameterize the compressions $C(\widetilde{A})$ by means of abstract boundary conditions. This enables us to characterize various properties of $C(\widetilde{A})$ (in particular, self-adjointness) in terms of the parameter for $\widetilde{A}$ in the Krein formula for resolvents. We describe also the compressions of a certain class of finite-codimensional extensions. The applications to eigenfunction expansions of differential operators are specified.

The above results develop the results by A. Dijksma and H. Langer obtained for a densely defined symmetric operator $A$ with finite and equal deficiency indices.

**References**

[1] V.I. Mogilevskii, *On compressions of self-adjoint extensions of a symmetric linear relation*, Integr. Equ. Oper. Theory **91:9** (2019).

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Eigenvalues of Graphs

*Mohr Samuel* `Friday 17’45 (ZS 1)`

A *graph* is a combinatorical object consisting of a finite set of vertices and edges, which connect vertices. Graphs can be represented as matrices, therefore, we can use various tools of algebra. The *eigenvalues* of a graph are the eigenvalues of a representing matrix and the *spectrum* of a graph is the set of its eigenvalues. Several graph properties can be easily described by its eigenvalues.

In this talk, a very short introduction to spectral graph theory is given and some results on the spectra of graphs are presented.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Equivalence between the complex-rotation and scattering-matrix resonances in the Friedrichs-Faddeev model

*K. Motovilov Alexander* `Thursday 15’30 (ZS 3)`

Among various understandings of the term “resonance” in quantum mechanics, the two most common interpretations are as follows. (1) Resonance is a complex energy value producing a pole to the scattering matrix analytically continued to the so-called unphysical energy sheet(s). (2) Resonance is a complex eigenvalue of the complexly deformed Hamiltonian under consideration. In the present work, we restrict ourselves to the study of the Friedrichs-Faddeev model. For this model, we prove that, under a hypothesis adopted, the resonances understood in the senses (1) and (2) are equivalent.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Optimal control of parabolic equations using spectral calculus

*Nakic Ivica* `Saturday 11’45 (HS 5)`

Let $\mathcal{H}$ be a Hilbert space and $A$ be a lower-bounded self-adjoint operator in $\mathcal{H}$. We consider for $f \in L_2(\mathbb{R};\mathcal{H})$ and $u \in \mathcal{H}$ the Cauchy problem

$$\begin{cases} y'(t) + Ay(t) = f(t) & \text{for } t \geq 0, \\ y(0) = u. \end{cases}$$

For $\epsilon, T > 0$ and $y^* \in \mathcal{H}$ we introduce the optimal control problem

$$\min_{u\in\mathcal{H}} \{J(u) \colon \|y(T) - y^*\| \leq \epsilon\}$$

where

$$J(u) = \frac{\alpha}{2}\|u\|^2 + \frac{1}{2}\int_0^T \beta(t)\|y(t) - w(t)\|^2 \mathrm{d}t,$$

$\alpha > 0$, $\beta \in L^\infty((0,T);[0,\infty))$, and $w \in L^2((0,T);\mathcal{H})$.

We show how to solve this problem using spectral calculus of the operator $A$ and propose an efficient numerical method for calculating the solution.

The talk is based on joint work with L. Grubišić, M. Lazar and M. Tautenhahn.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Self-adjoint extensions of infinite quantum graphs

*Nicolussi Noema* `Friday 15'30 (ZS 1)`

In the last decades, quantum graphs (Laplacians on metric graphs) have become popular objects of study and the analysis of spectral properties relies on the self-adjointness of the Laplacian. Whereas on finite metric graphs the Kirchhoff Laplacian is always self-adjoint, much less is known about the self-adjointness problem for graphs having infinitely many edges and vertices. Intuitively the question is closely related to finding appropriate boundary notions for infinite graphs.
In this talk we study the connection between self-adjoint extensions and the notion of graph ends, a classical graph boundary introduced independently by Freudenthal and Halin. Our discussion includes a lower estimate on the deficiency indices and a geometric characterization of uniqueness of a Markovian extension of the minimal Kirchhoff Laplacian.

Based on joint work with Aleksey Kostenko (Ljubljana & Vienna) and Delio Mugnolo (Hagen).

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## The semi-classical limit with delta potentials

*Posilicano Andrea* `Saturday 15'00 (ZS 1)`

We consider the semi-classical limit of the quantum evolution of Gaussian coherent states whenever the Hamiltonian $H$ is given, as sum of quadratic forms, by $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} \dot{+} \alpha\delta_0$, with $\alpha \in \mathbb{R}$ and $\delta_0$ the Dirac delta-distribution at $x = 0$. We show that the quantum evolution can be approximated, uniformly for any time away from the collision time and with an error of order $\hbar^{3/2-\lambda}$, $0 < \lambda < 3/2$, by the quasi-classical evolution generated by a self-adjoint extension of the restriction to $C_c^\infty(M_0)$, $M_0 := \{(q,p) \in \mathbb{R}^2 \,|\, q \neq 0\}$, of ($-i$ times) the generator of the free classical dynamics; such a self-adjoint extension does not correspond to the classical dynamics describing the complete reflection due to the infinite barrier. Similar approximation results are also provided for the wave and scattering operator.

This is a joint work with Claudio Cacciapuoti and Davide Fermi.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Semigroups for integro-differential equations with convolution memory terms

*Rautian Nadezhda* `Friday 17'15 (ZS 3)`

We discuss an abstract evolution equation with memory arising in linear viscoelasticity, presenting the results based on the classical approaches stated in the monographs [1], [2]. These results can be easily extended and adapted to many other differential models containing memory terms in convolution form.

We reduce the initial-boundary value problem for this equation to the Cauchy problem for differential equation of the first oder in separable Hilbert space. We prove the existence of a contraction semigroup and establish exponential stability within standard assumptions on the memory kernels. On the base of these results, we prove the theorem about the strong solvability of the appropriate initial boundary-value problem. Moreover, we consider some examples for exponential and fractional-exponential kernels (Rabotnov functions) (see [3]).

**References**

[1] K.J. Engel, R. Nagel One-Parameter Semigroups for Linear Evolution Equations. Springer-Verlag, New York, 2000.

[2] Amendola G., Fabrizio M., Golden J. M. Thermodynamics of Materials with memory. Theory and applications. Springer New-York - Dordrecht - Heidelberg - London, 2012

[3] Vlasov V. V., Rautian N. A. Well-posed solvability and the representation of solutions of integro-differential equations arising in viscoelasticity // Differential Equations. 2019. Vol. 55, no. 4. P. 561-574.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Theorem of Hermite-Biehler for matrix-valued entire functions

*Reiffenstein Jakob* `Friday 16'00 (ZS 1)`

An entire function $E$ belongs to the Hermite-Biehler class HB if $E$ has no real zeros, and the inequality $|E(\overline{z})| < |E(z)|$ holds for every $z \in \mathbb{C}_+$. Suppose $E = A + iB$ where $A$ and $B$ are real and entire. Then the theorem of Hermite-Biehler states that - in essence - the function $E$ is of class HB iff all zeroes of $A$ and $B$ are real, simple, and interlace - or, equivalently, iff $\frac{A}{B}$ is a Herglotz function. The goal is to present a version of this theorem for matrix-valued entire functions together with a generalized interlacing property. In particular, we study the pattern of zeroes and poles of matrix-valued Herglotz functions having a continuation to $\mathbb{C}$ that is meromorphic and real. In fact, the following statement holds: A real $Q \in \mathcal{M}(\mathbb{C}, \mathbb{C}^{n\times n})$ is Herglotz iff the determinants of its principal submatrices all satisfy a suitable interlacing property. We can then define a matrix-valued Hermite-Biehler class and apply this result to the matrix-valued Herglotz function $B^{-1}A$ to obtain a version of the Hermite-Biehler theorem for matrix-valued entire functions.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Canonical systems in ideals of compact operators

*Romanov Roman* `Thursday 10'30 (HS 5)`

We characterise the Hamiltonians of canonical systems with resolvents from a wide class of ideals of compact operators containing the trace class. In particular, we characterize the systems with discrete spectrum answering a question of Louis de Branges. The talk is based on a joint work with Harald Woracek.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Discrete Dirac system and Arov–Krein entropy

*Sakhnovich Alexander* `Saturday 17'15 (ZS 3)`

Self-adjoint discrete Dirac systems have many analogies with the famous Szegő recurrences from the theory of orthogonal polynomials. In particular, Verblunsky-type coefficients and Verblunsky-type theorems appear in the theory of discrete Dirac systems. The asymptotics of the fundamental solutions of the discrete Dirac systems is closely connected with the results on Arov–Krein entropy. We will also discuss the analogs of the Arov–Krein entropy in the case of indefinite metrics. The talk is mostly based on the papers [1, 2].

**References**

[1] I. Roitberg and A.L. Sakhnovich, Arov–Krein entropy functionals and indefinite interpolation problems, Integr. Equ. Oper. Theory **91**:50 (2019), https://doi.org/10.1007/s00020-019-2549-8.

[2] A.L. Sakhnovich, New "Verblunsky-type" coefficients of block Toeplitz and Hankel matrices and of corresponding Dirac and canonical systems, J. Approx. Theory **237** (2019), 186–209.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## From input-to-state stability to semigroup perturbations

*Schwenninger Felix* `Sunday 12'15 (HS 5)`

In this talk recent relations between stability concepts in control theory, the Desch-Schappacher perturbation result on semigroups and Baillon's result on maximal regularity will be discussed. The geometry of the underlying Banach space is crucial here.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## On embedding constants in Sobolev spaces. Application to spectral problems with coefficients-distributions.

*Sheipak Igor* `Thursday 15'00 (ZS 3)`

We study spectral properties of boundary value problem

$$(-1)^n y^{(2n)} = \lambda\langle\delta^{(k)}(x-a), y\rangle\delta^{(k)}(x-a),$$

$$y^{(j)}(0) = y^{(j)}(1) = 0, \quad j = 0, 1, \ldots, n-1.$$

Let us consider the problem of finding the smallest eigenvalue $\lambda_{min}$ in a parameter $a$. This problem has close links with embedding problems of Sobolev spaces

$$\mathring{W}_2^n[0,1] \hookrightarrow \mathring{W}_\infty^k[0,1], \quad 0 \le k \le n-1.$$

We describe properties of functions $A_{n,k}^2$ which provides an accurate estimate in the inequality

$$|f^{(k)}(a)|^2 \le A_{n,k}^2(a) \int_0^1 |y^{(n)}(x)|^2\, dx.$$

Let us denote $\Lambda_{n,k}^2 = \max_{a\in[0;1]} A_{n,k}^2(a)$.

We've proved that the point of the global maximum of function $A_{n,k}^2$ is the closest maximum point of the function $A_{n,k}^2$ to the middle point of the interval $[0;1]$. We also show that $\Lambda_{n,k}^2$ — the value of the function $A_{n,k}^2$ at the global maximum point is a square of precise embedding constants. The relationship to the lowest eigenvalue of the problem is determined by the formula $\lambda_{min} = \Lambda_{n,k}^{-2}$. We also obtained the formula for $\Lambda_{n,k}^2$ for all even $k$.

The work is supported by Grant of the President of the Russian Federation for the Program of supporting of "Leading Scientific Schools", project NSh. 6222.2018.1.

— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —

## Wave model of symmetric operators

*Simonov Sergey* `Thursday 16'45 (ZS 3)`

We will discuss ways and methods to construct a new functional model of symmetric operators, the *wave model*, along with examples of such constructions for particular classes of differential operators. This model was proposed by M. I. Belishev in 2013 on a heuristic level in an attempt to find a universal abstract scheme for solving inverse problems with the boundary control (BC) method. The talk is based on joint works with M. I. Belishev.

— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —

## On evolution semigroups and Trotter product operator-norm estimates

*Stephan Artur* `Friday 15'30 (ZS 3)`

Evolution semigroups and Trotter products have been an important part of Hagen's scientific research over the past 40 years: his dissertation on non-autonomous Cauchy problems explains the one-to-one correspondence between their solution operators (or propagators) and evolution semigroups on curves in a Banach space. Later Hagen made several contributions to improving operator-norm estimates for the convergence of Trotter- and Trotter-Kato products. In recent years, Hagen Neidhardt, Valentin Zagrebnov and myself were able to link evolution equations with the Trotter product formula to derive explicit convergence rate estimates for approximations of the solution operator [1, 2, 3]. In my talk I give an overview on these convergence rate estimates focusing more on the Banach space setting. The talk is based on joint works with Hagen Neidhardt and Valentin Zagrebnov, and is also quite related to the talk of Valentin who, in contrast, considers evolution equations on Hilbert spaces.

### References


[1] H. Neidhardt, A. Stephan and V. A. Zagrebnov, Convergence rate estimates for approximations of solution operators. accepted for publication in *PRIMS*, 2019.

[2] ———, Remarks on the operator-norm convergence of the Trotter product formula, published in *IEOT*, 2018.

[3] ———, Operator-Norm Convergence of the Trotter Product Formula on Hilbert and Banach Spaces: A Short Survey, published in *Current Research in Nonlinear Analysis*, SOIA, 2018.


– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Everything is possible for the domain intersection of an operator and its adjoint

*Tretter Christiane* `Thursday 10'00 (HS 5)`

In this talk it will be shown that even for very nice classes of linear operators, including maximal sectorial operators $T$, everything is possible for the domain intersection $\mathrm{dom}T \cap \mathrm{dom}T^*$, even the most extreme case $\mathrm{dom}T \cap \mathrm{dom}T^* = \{0\}$.
(*joint work with Yury Arlinskii*)

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Spectral analysis and representation of solutions of Volterra integro-differential equations with fractional exponential kernels

*Vlasov Victor* `Friday 17'45 (ZS 3)`

We study integro-differential equations with unbounded operator coefficients in Hilbert space. The equations under consideration are abstract hyperbolic equations perturbed by terms containing Volterra integral operators. The kernels of these Volterra operators are sums of fractional exponential Rabotnov functions (see [1]). These integro-differential equations can be realized as partial integro-differential equations arising in the theory of viscoelasticity (see [1]) and also as GurtinPipkin integro-differential equations (see [2]), which describe heat transfer with a finite rate in media with memory.

We establish the existence of strong and generalized solutions of the above integro-differential equations and the spectral analysis of operator functions being symbols of these equations is performed. This makes it possible to obtain representations and estimates of solutions of the equations in question (see [3]).

### References


[1] Yu. N. Rabotnov, Elements of Hereditary Solid Mechanics (Nauka, Moscow, 1977; Mir, Moscow, 1980).

[2] M. E. Gurtin and A. C. Pipkin, Arch. Ration. Mech. Anal. 1968, Vol.31 (2), P. 113-126.

[3] Vlasov V. V., Rautian N. A. Well-posed solvability and the representation of solutions of integro-differential equations arising in viscoelasticity // Differential Equations. 2019. Vol. 55, no. 4. P. 561-574.


– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## Factorizations, invariant subspaces and multi-valency

*Wietsma Rudi* `Thursday 15’30 (ZS 1)`

It is shown how the factorization of scalar generalized Nevanlinna functions, the (Krein-Langer) factorization of generalized Schur functions, the invariant subspace property of selfadjoint relations in Pontryagin spaces and the invariant subspace property of contractive operators in Pontryagin spaces are all essentially equivalent. To establish these connections the concept of multi-valency is a central tool. The concept of multi-valency not only provides new characterizations for the mentioned classes of functions and easier proofs for the afore-mentioned properties, but it also explains the fundamental difference between the factorization of (scalar) generalized Nevanlinna functions and the factorization of (scalar) generalized Schur functions.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## The angle along a curve and range-kernel complementarity

*Yannakakis Nikos* `Saturday 17’15 (ZS 1)`

Let $X$ be a Banach space and $A \in B(X)$. Range-kernel complementarity, i.e. the decomposition

$$X = R(A) \oplus N(A)\,,$$

stands right next to the invertibility of $A$, since if it holds then $A$ is of the form “invertible $\oplus\, 0$”.

As it is well-known, in finite dimensions range-kernel complementarity is equivalent to $R(A) \cap N(A) = \{0\}$ which in turn is equivalent to the ascent (the length of the null-chain) of $A$ being less than or equal to one. In infinite dimensions things are significantly different as $R(A) \cap N(A) = \{0\}$ is no longer sufficient and one needs the additional assumption that $R(A) = R(A^2)$. Note that the latter is equivalent to the descent (the length of the range chain) of $A$ being less than or equal to one.

In this talk we define the angle of a bounded linear operator $A$, along an unbounded curve emanating from the origin and use it to characterize range-kernel complementarity. In particular we show that if 0 faces the unbounded component of the resolvent set, then $X = R(A) \oplus N(A)$ if and only if $R(A)$ is closed and some angle of $A$ is less than $\pi$.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –

## The Howland-Evans-Neidhardt approach to approximation of propagators

*Zagrebnov Valentin A.* `Friday 12’45 (HS 5)`

My talk is devoted to evolution equations of the form

$$\frac{\partial}{\partial t}u(t) = -(A + B(t))u(t), \quad u(t) \in \mathfrak{H} \text{ for } t \in (0,T),$$

in a separable Hilbert space $\mathfrak{H}$. Here $A$ is a positive self-adjoint operator and $B(\cdot)$ is family of positive self-adjoint operators such that $\mathrm{dom}(A^\alpha) \subseteq \mathrm{dom}(B(t))$ for some $\alpha \in [0,1)$ and the map $t \mapsto A^{-\alpha}B(t)A^{-\alpha}$ is operator-norm Hölder continuous in $(0,T)$ with exponent $\beta \in (0,1)$. It is shown that the solution operator (*propagator*) $U(t,s)$ of the evolution equation can be approximated in the operator norm by the *product formula* for *approximants* $\{U_n(t,s)\}_{n\geq 1}$ that involves semigroups generated by $A$ and $B(t)$ provided the condition $\beta > 2\alpha - 1$ is satisfied. The rate of convergence of $\{U_n(t,s)\}_{n\geq 1}$ to $U(t,s)$ is defined by the Hölder exponent $\beta$ and has the order $O(1/n^\beta)$ [NSZ]. The result is proved using the Howland-Evans-Neidhardt approach to construction of the *evolution* semigroups and of the *approximants* of propagators.

**References**

[NSZ] H. Neidhardt, A. Stephan, and V. A. Zagrebnov. *Trotter Product Formula and Linear Evolution Equations on Hilbert Spaces*. Analysis and Operator Theory. Dedicated in Memory of Tosio Kato’s 100th Birthday, Springer vol.146, Berlin 2019, pp. 271–299.

– – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – – –