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We study spectral properties of boundary value problem

\[ (-1)^n y^{(2n)} = \lambda (\delta^{(k)}(x-a), y)\delta^{(k)}(x-a), \]
\[ y^{(j)}(0) = y^{(j)}(1) = 0, \quad j = 0, 1, \ldots, n-1. \]

Let us consider the problem of finding the smallest eigenvalue \( \lambda_{\text{min}} \) in a parameter \( a \). This problem has close links with embedding problems of Sobolev spaces

\[ W^{2}_2[0, 1] \hookrightarrow W^{k}_\infty[0, 1], \quad 0 \leq k \leq n-1. \]

We describe properties of functions \( A^{2}_{n,k} \) which provides an accurate estimate in the inequality

\[ |f^{(k)}(a)|^2 \leq A^{2}_{n,k}(a) \int_0^1 |y^{(n)}(x)|^2 dx. \]

Let us denote \( \Lambda^{2}_{n,k} = \max_{a \in [0; 1]} A^{2}_{n,k}(a) \).

We’ve proved that the point of the global maximum of function \( A^{2}_{n,k} \) is the closest maximum point of the function \( A^{2}_{n,k} \) to the middle point of the interval \([0; 1] \). We also show that \( \Lambda^{2}_{n,k} \) — the value of the function \( A^{2}_{n,k} \) at the global maximum point is a square of precise embedding constants. The relationship to the lowest eigenvalue of the problem is determined by the formula \( \lambda_{\text{min}} = \Lambda^{2}_{n,k} \).

We also obtained the formula for \( \Lambda^{2}_{n,k} \) for all even \( k \).

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