Canonical systems whose Weyl coefficients have regularly varying asymptotics

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For a two-dimensional canonical system $y'(t) = zJH(t)y(t)$ on the half-line $(0, \infty)$ whose Hamiltonian $H$ is positive semi-definite a.e., let $q_H$ be its Weyl coefficient. De Branges’ inverse spectral theorem states that the assignment $H \mapsto q_H$ is a bijection from trace-normed Hamiltonians onto the set of Nevanlinna functions. In this talk I shall answer the question when $q_H(ir) \sim i\omega a(r)$ as $r \to \infty$ where $\omega \in \mathbb{C} \setminus \{0\}$ and $a$ is a regularly varying function, i.e. $\exists \alpha \in \mathbb{R}$ such that $\lim_{r \to \infty} \frac{a(\lambda r)}{a(r)} = \lambda^\alpha$ for all $\lambda > 0$. Note that the class of regularly varying functions includes, e.g. $a(r) = r^\alpha (\log r)^{\beta_1} (\log \log r)^{\beta_2}$ with $\alpha, \beta_1, \beta_2 \in \mathbb{R}$ but also some oscillating functions. I shall also discuss the relation between $\omega$ and $a$ on one hand and properties of $H$ on the other hand. The talk is based on joint work with Raphael Pruckner and Harald Woracek.