Geometric approximations of point interactions

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In this talk we address the problem of an approximation of some singular Schrödinger operators describing the motion of a particle in a potential being supported at a discrete set. These operators are known as solvable models in quantum mechanics; the word solvable reflects the fact that their mathematical and physical quantities (spectrum, eigenfunctions, etc.) can be determined explicitly. Such models are also called point interactions.

One of the main problems arising in the theory of solvable models is their approximations by more “realistic” ones. In the talk we address the question of approximation of the so-called $\delta$ and $\delta'$-interactions using geometrical tools, namely, the Neumann Laplacians on thin domains with waveguide geometry. For the underlying operators we establish (a kind of) norm resolvent convergence and the Hausdorff convergence of their spectra. To approximated $\delta$-interactions we use waveguides with attached “room-and-passage” bumps, while for $\delta'$-interactions we utilize waveguides consisting of two thin straight tubular domains connected through a tiny window.