De Branges-Pontryagin spaces and embedding of de Branges matrices with negative squares in generalized J-inner matrices

Volodymyr Derkach

The notion of entire de Branges matrix $\mathcal{E}(\lambda)$ with negative squares $\kappa$ is introduced. Associated to such matrix is a de Branges-Pontryagin spaces $\mathcal{B}(\mathcal{E})$ with negative index $\kappa$. The problem of embedding of de Branges matrix $\mathcal{E}(\lambda)$ with negative squares in generalized J-inner matrix $A(\lambda)$ is considered. This problem is proved to be solvable when the space $\mathcal{B}(\mathcal{E})$ is invariant under the generalized backward shift operator. The theory of rigged de Branges-Pontryagin spaces is developed and then applied to obtain a solution of this embedding problem. A formula for factoring an arbitrary generalized J-inner entire matrix valued function into the product of a singular factor and a perfect one is found analogous to the known factorization formulas for J-inner matrix valued functions.