# problem sheet 4

discussion: week of Monday, 28.10.2019

**4.1.** Let $C$, $\alpha > 0$ and consider the function $h \mapsto f(h) = Ch^\alpha$. Why is this function a straight line in a `loglog`-plot? What is its slope? Other popular plotting schemes are, `semilogx` and `semilogy`. Which one would you use to plot functions of the form $N \mapsto Ce^{-bN}$? How would you proceed if you suspect that a function $h \mapsto f(h)$ has the form $f(h) = Ce^{-b/h}$?

**4.2.** We wish to show that the extrapolation of the composite trapezoidal rule is the composite Simpson rule. To that end, let $T(h)$ be the composite trapezoidal rule with step size $h = (b-a)/N$ and $S(h)$ be the composite Simpson rule with step size $h = (b-a)/N$. Use Romberg extrapolation with step sizes $h_i = (b-a)2^{-i}$, $i = 0, 1, \ldots,$

**a)** Extrapolation of the composite trapezoidal rule (with step size $h$) has in column $m = 0$ the values $T(h_i)$. Show: in column $m = 1$ of the Neville scheme are the values

$$N_i := T(h_{i+1}) + \frac{1}{3}(T(h_{i+1}) - T(h_i))$$

**b)** Show: $N_i = S(h_i)$.

**4.3.** Write a program that realizes Romberg extrapolation of the composite trapezoidal rule for step sizes $h_i = (b-a)2^{-i}$, $i = 0, 1, \ldots$, (i.e. use your program of Exercise 3.3 with $N = 2^i$ and use the Neville scheme for the extrapolation). Plot, using `loglog`, the error for the values in the columns $m = 0$, $m = 1$, $m = 2$ of the Neville scheme for the integrands

$$f_1(x) = x^{0.2}, \quad f_2(x) = x^{10}, \quad f_3(x) = x^2$$

and the integration region $[0, 1]$. What do you observe?

**4.4.** Write a program with signature $I =$ `adapt`$(f, a, b, \tau, h_{min})$ that realizes an adaptive quadrature for $\int_a^b f(x)\,dx$. The quadrature should be based on the Simpson rule. $\tau$ is the desired (absolute) accuracy and $h_{min}$ the minimal interval length. To estimate the accuracy, compare the value of the Simpson rule $S_{\{a,b\}}(f)$ for the integration on $[a, b]$ with the value $S_{\{a,m\}}(f) + S_{\{m,b\}}(f)$ with $m = (a+b)/2$. Use your algorithm for the integration of the function

$$\begin{cases} \frac{1}{2}e^x & x < 1/3 \\ e^x & x \geq 1/3 \end{cases}$$

over $[0, 1]$. Use $\tau = h_{min} = 2^{-j}$, $j = 0, \ldots, 10$. Plot the error versus $\tau$. What convergence do you observe? Why was $h_{min} = \tau$ chosen?