

# numerical differentiation with one-sided difference quotients

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$$D(h) = \frac{f(x_0 + h) - f(x_0)}{h}$$

$h$	$m = 0$	$m = 1$	$m = 2$	$m = 3$
$h_0$	$D(h_0) = D_{00}$	$D_{01}$	$D_{02}$	$D_{03}$
$h_1$	$D(h_1) = D_{10}$	$D_{11}$	$D_{12}$	$D_{13}$
$h_2$	$D(h_2) = D_{20}$	$D_{21}$	$D_{22}$	$D_{23}$
$h_3$	$D(h_3) = D_{30}$	$D_{31}$	$D_{32}$	$D_{33}$
$h_4$	$D(h_4) = D_{40}$	$D_{41}$	$D_{42}$	$\vdots$
$h_5$	$D(h_5) = D_{50}$	$D_{51}$	$\vdots$	
$h_6$	$D(h_6) = D_{60}$	$\vdots$		
$\vdots$	$\vdots$			

$$D_{i0} = D(h_i)$$

$$D_{ij} = D_{(i+1)(j-1)} - \frac{h_{i+j}}{h_{i+j} - h_i} [D_{(i+1)(j-1)} - D_{i(j-1)}], \quad j \geq 1$$

## numerical differentiation of $f(x) = |x|^{3/2}$ at $x = 0$

extrapolation error for one-sided difference quotients

$h$	m=0	m=1	m=2	m=3	m=4	m=5	m=6	m=7
$2^0$	1.00 <sub>0</sub>	4.14 <sub>-1</sub>	2.52 <sub>-1</sub>	1.68 <sub>-1</sub>	1.15 <sub>-1</sub>	8.06 <sub>-2</sub>	5.66 <sub>-2</sub>	3.99 <sub>-2</sub>
$2^{-1}$	7.07 <sub>-1</sub>	2.93 <sub>-1</sub>	1.79 <sub>-1</sub>	1.19 <sub>-1</sub>	8.17 <sub>-2</sub>	5.70 <sub>-2</sub>	4.00 <sub>-2</sub>	2.82 <sub>-2</sub>
$2^{-2}$	5.00 <sub>-1</sub>	2.07 <sub>-1</sub>	1.26 <sub>-1</sub>	8.40 <sub>-2</sub>	5.77 <sub>-2</sub>	4.03 <sub>-2</sub>	2.83 <sub>-2</sub>	
$2^{-3}$	3.54 <sub>-1</sub>	1.46 <sub>-1</sub>	8.93 <sub>-2</sub>	5.94 <sub>-2</sub>	4.08 <sub>-2</sub>	2.85 <sub>-2</sub>		
$2^{-4}$	2.50 <sub>-1</sub>	1.04 <sub>-1</sub>	6.31 <sub>-2</sub>	4.20 <sub>-2</sub>	2.89 <sub>-2</sub>			
$2^{-5}$	1.77 <sub>-1</sub>	7.32 <sub>-2</sub>	4.46 <sub>-2</sub>	2.97 <sub>-2</sub>				
$2^{-6}$	1.25 <sub>-1</sub>	5.18 <sub>-2</sub>	3.16 <sub>-2</sub>					
$2^{-7}$	8.84 <sub>-2</sub>	3.66 <sub>-2</sub>						
$2^{-8}$	6.25 <sub>-2</sub>							
error	$\sqrt{h}$	$\sqrt{h}$	$\sqrt{h}$	$\sqrt{h}$	$\sqrt{h}$	$\sqrt{h}$		

reason for “failure” of extrapolation: The function  $D(h) = h^{-1}(|h|^{3/2} - 0) = \sqrt{|h|}$  is not “smooth” at  $h = 0$ .