Neville scheme

$$p_{i,0}(x) = f_i \qquad i = 0, \ldots, n$$

$$p_{i,m}(x) = \frac{(x - x_i)p_{i+1,m-1}(x) - (x - x_{i+m})p_{i,m-1}(x)}{x_{i+m} - x_i} \quad , i = 0, \ldots, n-m,\ m = 1, \ldots, n$$

$$\begin{array}{c|ccccccccccc}
x_0 & f_0 & =: & p_{0,0}(x) & \longrightarrow & p_{0,1}(x) & \longrightarrow & p_{0,2}(x) & \longrightarrow & \ldots & \longrightarrow & p_{0,n}(x) = p(x) \\
 & & & & \nearrow & & \nearrow & \vdots & & & \nearrow & \\
x_1 & f_1 & =: & p_{1,0}(x) & \longrightarrow & p_{1,1}(x) & & \vdots & & & & \\
 & & & & \nearrow & \vdots & & \vdots & & & & \\
x_2 & f_2 & =: & p_{2,0}(x) & & \vdots & & \vdots & & & & \\
\vdots & \vdots & & \vdots & & \vdots & & \vdots & & & & \\
\vdots & \vdots & & \vdots & & \vdots & \longrightarrow & p_{n-2,2}(x) & & & & \\
\vdots & \vdots & & \vdots & & \vdots & \nearrow & & & & & \\
\vdots & \vdots & & \vdots & \longrightarrow & p_{n-1,1}(x) & & & & & & \\
\vdots & \vdots & & \vdots & \nearrow & & & & & & & \\
x_n & f_n & =: & p_{n,0}(x) & & & & & & & &
\end{array}$$

Neville Scheme (example)

given:

$$(x_0, f_0) = (0, 1) \qquad (x_1, f_1) = (1, 3) \qquad (x_2, f_2) = (3, 2)$$

sought: value of interpolating polynomial at $x = 2$

starting values of Neville scheme (1. column):

$$p_{0,0}(2) = f_0 = 1$$
$$p_{1,0}(2) = f_1 = 3$$
$$p_{2,0}(2) = f_2 = 2$$

second step (2. column):

$$p_{0,1}(2) = \frac{(2-0)p_{1,0}(2) - (2-1)p_{0,0}(2)}{1-0} = \quad 2 \cdot 3 - 1 \cdot 1 = 5$$

$$p_{1,1}(2) = \frac{(2-1)p_{2,0}(2) - (2-3)p_{1,0}(2)}{3-1} = \frac{1 \cdot 2 - (-1) \cdot 3}{2} = \frac{5}{2}$$

result (3. column):

$$\boxed{p_{0,2}(2)} = \frac{(2-0)p_{0,1}(2) - (2-3)p_{1,1}(2)}{3-0} = \frac{2 \cdot \frac{5}{2} - (-1) \cdot 5}{3} = \boxed{\frac{10}{3}}$$