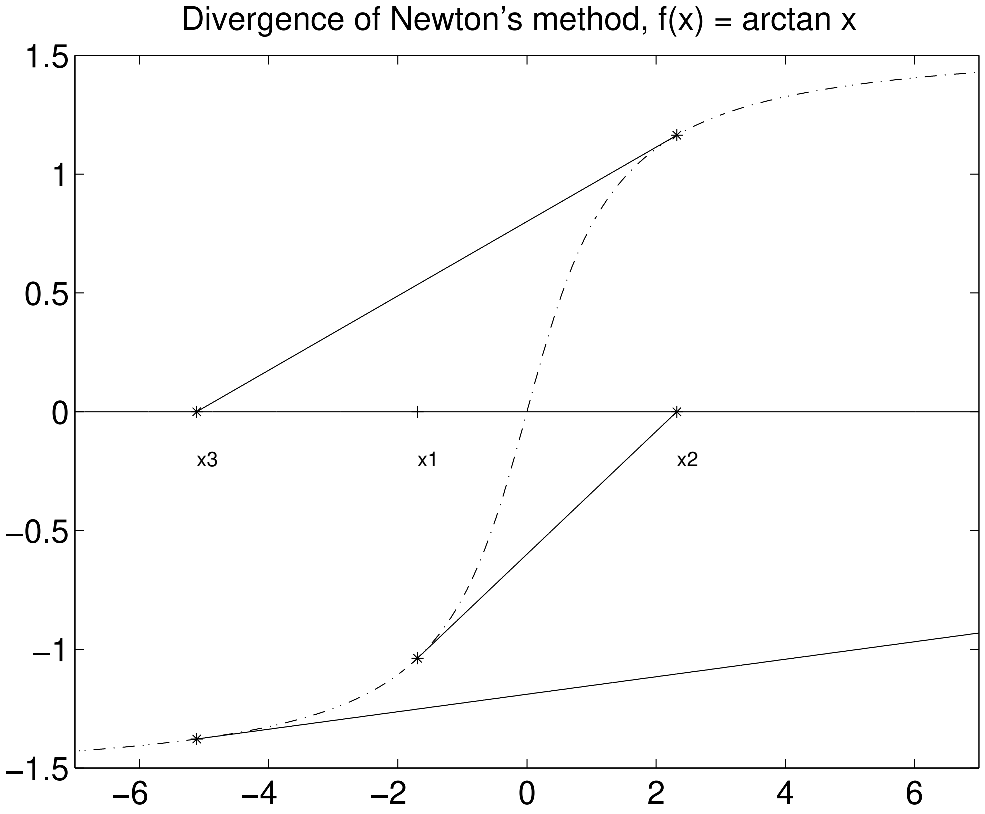
goal: compute zero of $f(x) = \textbf{arctan}\, x$

Newton's method: $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

n	x_n
0	1.5
1	-1.694
2	2.321
3	-5.114
4	32.296
5	$-1.575 \cdot 10^3$
6	$3.895 \cdot 10^6$
7	$-2.383 \cdot 10^{13}$



problem: "correction" $\frac{f(x_n)}{f'(x_n)}$ "overshoots" the zero $x^* = 0$. We have: $|x_{n+1}| > |x_n|$ for all n.

damped Newton method

solution:

damped Newton method: $$x_{n+1} = x_n - \lambda \frac{f(x_n)}{f'(x_n)}, \qquad \lambda \in (0,1)$$

n	λ	x_n
0	0.9	1.5
5	0.9	-1.375
10	0.9	$2.769 \cdot 10^{-4}$
15	0.9	$2.769 \cdot 10^{-9}$
20	0.9	$3.0 \cdot 10^{-14}$

n	λ	x_n
0	0.1	10
100	0.1	-4.858
200	0.1	$-2.700 \cdot 10^{-5}$
300	0.1	$-7.173 \cdot 10^{-10}$
400	0.1	$-2.0 \cdot 10^{-14}$

observation: convergence of the method even for x_0 far from $x^* = 0$ if the damping parameter λ is chosen suitable. However: convergence is only linear.

damped Newton method with damping parameter control

input: initial guess x_0, parameter μ, $q \in (0,1)$

```
λ₀ := 1,    n := 0
while ( stopping criterion not satisfied ){
    p := (f'(x_n))^{-1} f(x_n)
    while ( ||f(x_n)||_2^2 - ||f(x_n - λ_n p)||_2^2 < μ λ_n ||f(x_n)||_2^2 ) {
        λ_n := q λ_n
    }
    x_{n+1} := x_n - λ_n p
    λ_{n+1} := min{1, λ_n/q}
    n := n + 1
}
```

$\lambda_0 := 1, \quad n := 0$

`while ( stopping criterion not satisfied ){`

$\quad p := (f'(x_n))^{-1} f(x_n)$

$\quad$ `while` $\left(\|f(x_n)\|_2^2 - \|f(x_n - \lambda_n p)\|_2^2 < \mu \lambda_n \|f(x_n)\|_2^2 \right)$ `{`

$\qquad \lambda_n := q\lambda_n$

$\quad$ }

$\quad x_{n+1} := x_n - \lambda_n p$

$\quad \lambda_{n+1} := \min\{1, \lambda_n / q\}$

$\quad n := n + 1$

}

example: $f(x) = \arctan x, \quad x_0 = 1000, \quad q = \mu = 0.5$