Example of Lagrange interpolation

goal: find interpolating polynomial $p \in \mathcal{P}_2$ for the data

$$(0, 0), \quad (\frac{\pi}{4}, \frac{\sqrt{2}}{2}) \quad (\frac{\pi}{2}, 1)$$

solution:

$$p(x) = 0 \cdot \ell_0(x) + \frac{\sqrt{2}}{2} \cdot \ell_1(x) + 1 \cdot \ell_2(x),$$

$$\ell_0(x) = \frac{(x - \pi/4)(x - \pi/2)}{(0 - \pi/4)(0 - \pi/2)} = 1 - (1.909...)x + (0.8105...)x^2,$$

$$\ell_1(x) = \frac{(x - 0)(x - \pi/2)}{(\pi/4 - 0)(\pi/4 - \pi/2)} = (2.546...)x - (1.62...)x^2$$

$$\ell_2(x) = \frac{(x - 0)(x - \pi/4)}{(\pi/2 - 0)(\pi/2 - \pi/4)} = -(0.636...)x + (0.81...)x^2.$$

Hence, $p(x) = (1.164...)x - (0.3357...)x^2$

“application”

The data

$$(0,0), \quad (\frac{\pi}{4}, \frac{\sqrt{2}}{2}) \quad (\frac{\pi}{2}, 1)$$

were chosen as $f_i = \sin(x_i)$, i.e., $f(x) = \sin x$.

- An approximation to $f'(0) = 1$ could be taken to be $f'(0) \approx p'(0) = 1.164...$
- An approximation to $\int_0^{\pi/2} f(x)\,dx = 1$ is $\int_0^{\pi/2} p(x)\,dx = 1.00232...$