

comparison of solution methods

$$W = \begin{pmatrix} 1 & 0 & \dots & 0 & 1 \\ -1 & \dots & \dots & \vdots & \vdots \\ \vdots & \dots & \dots & 0 & \vdots \\ \vdots & & \dots & 1 & 1 \\ -1 & \dots & \dots & -1 & 1 \end{pmatrix} \in \mathbb{R}^{n \times n},$$

$$b = \frac{1}{n}(0, 1, 2, \dots, n-2, n)^\top,$$

$$x = \frac{1}{n}(-1, -1, \dots, -1, 1)^\top$$

		LU-factorization(Alg. I)			QR-factorization(Alg. II)		full pivoting (Alg. III)		
n	cond(A)	error	cond(L)	cond(U)	error	cond(R)	error	cond(L)	cond(U)
10	4.39	1.5 ₋₁₅	1.9 ₃	6.8 ₂	2.1 ₋₁₆	4.39	6.7 ₋₁₆	13.2	3.6
30	13.3	2.5 ₋₁₀	6.5 ₉	7.2 ₈	1.5 ₋₁₄	13.3	8.5 ₋₁₇	38.8	3.6
50	22.3	4.7 ₋₄	1.2 ₁₆	7.5 ₁₄	6.7 ₋₁₆	22.3	1.5 ₋₁₆	64.2	3.7

condition of the problem $b \mapsto A^{-1}b$ realized as:

Alg I:	$b \mapsto U^{-1}(L^{-1}b)$	partial conditionings:	$\text{cond}(U), \text{cond}(L)$
Alg II:	$b \mapsto R^{-1}(Q^\top b)$	partial conditionings:	$\text{cond}(Q^\top) = 1$ $\text{cond}(R) = \text{cond}(A)$
Alg III:	$b \mapsto Q^\top U^{-1} L^{-1} P b$	partial conditionings:	$\text{cond}(L), \text{cond}(U),$ $\text{cond}(Q^\top) = 1, \text{cond}(P) = 1$