

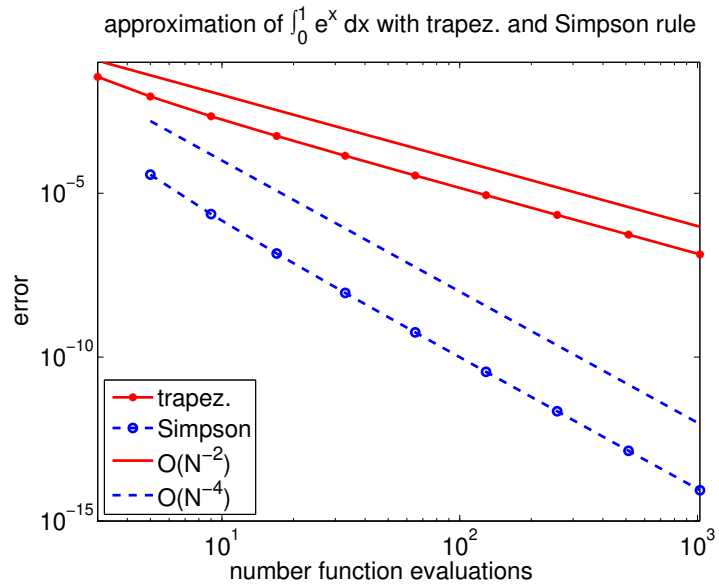
composite Newton-Cotes formulas (smooth integrands)

goal: compute $\int_0^1 e^x dx$ with composite trapezoidal and Simpson rule.

The errors E_{trap} , $E_{Simpson}$ satisfy (F denotes the number of function evaluations):

$$E_{trap}(h) \leq Ch^2 \sim CF^{-2}, \quad E_{Simpson} \leq Ch^4 \sim CF^{-4},$$

h	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	2^{-8}
$F_{trap} \sim 1/h$	2	3	5	9	17	33	65	129	257
E_{trap}	1.4 ₋₁	3.6 ₋₂	8.9 ₋₃	2.2 ₋₃	5.6 ₋₄	1.4 ₋₄	3.5 ₋₅	8.7 ₋₆	2.2 ₋₆
$F_{Simpson} \sim 1/h$	3	5	9	17	33	65	129	257	513
$E_{Simpson}$	5.8 ₋₄	3.7 ₋₅	2.3 ₋₆	1.5 ₋₇	9.1 ₋₉	5.7 ₋₁₀	3.6 ₋₁₁	2.2 ₋₁₂	1.4 ₋₁₃



conclusion: for smooth integrands it is more efficient, to employ higher order quadrature formulas.

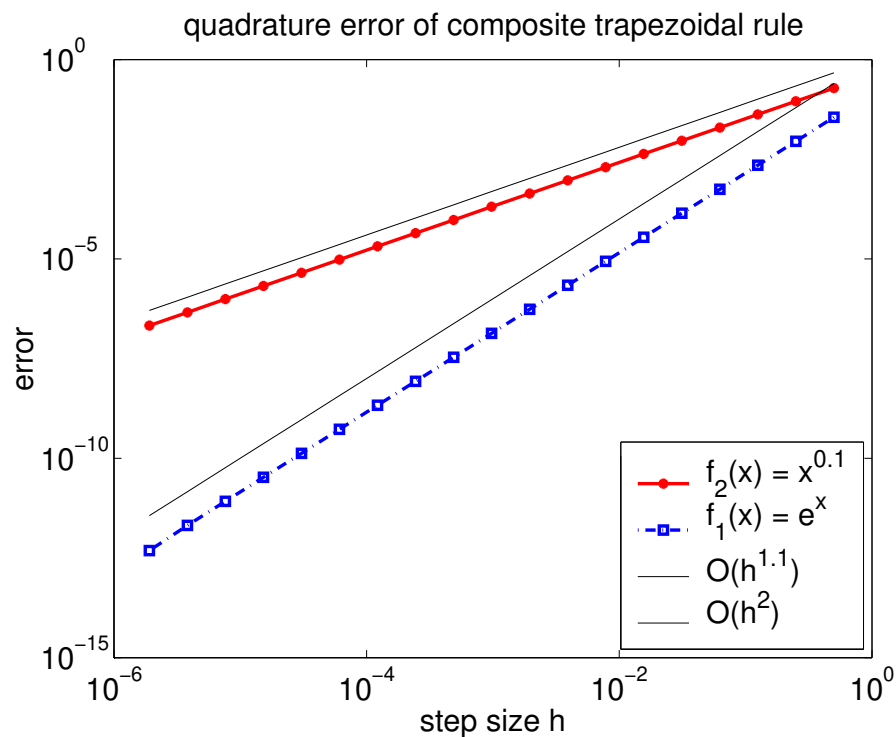
composite Newton-Cotes formulas (nonsmooth integrands)

$$f_1(x) := e^x, \quad f_2(x) := x^{0.1}$$

$E_i(h) :=$ quadrature error of composite trapezoidal rule for $\int_0^1 f_i(x) dx$.

$$f_1(x) = e^x \implies f_1 \in C^2([0, 1]) \implies E_1(h) \leq Ch^2,$$

$$f_2(x) = x^{0.1} \implies f_2 \notin C^2([0, 1]) \implies \text{cannot expect } E_2(h) \leq Ch^2$$



Remark: one can show that

$$E_2(h) \leq Ch^{1.1}$$