Multi-Scale Finite Element Method to Simulate Eddy Currents in Laminated Iron

Karl Hollaus and Joachim Schöberl

Institute for Analysis and Scientific Computing
Vienna University of Technology, Austria

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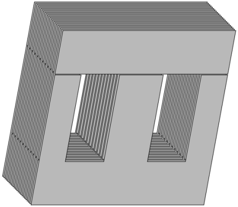
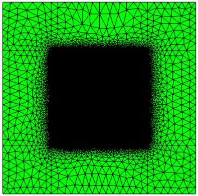
Motivation

Aim: Simulation of eddy current losses in large transformers

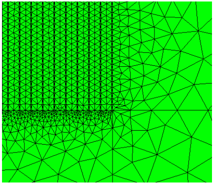
Large transformer



Laminated core



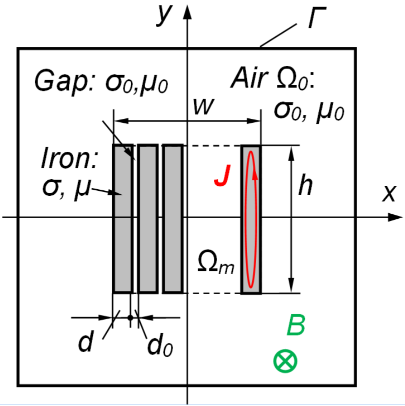
FE-Model with 100 laminates



Detail, lower right corner

Outline

Problem Description:



Draft of the problem in 2D

Conductivity:

σ ... iron
σ_0 ... air

Permeability:

μ ... iron
μ_0 ... air

Dimensions:

$d + d_0$ ... iron + air
$k_f = \frac{d}{d+d_0}$ ... fill factor

Quantities:

B ... Magnetic flux density
J ... Current density

Boundary Value Problem:

$$\begin{aligned} \operatorname{curl} \frac{1}{\mu} \operatorname{curl} \mathbf{A} + j\omega\sigma\mathbf{A} &= \mathbf{0} \quad \text{in } \Omega = \Omega_m \cup \Omega_0, \\ \mathbf{A} \times \mathbf{n} &= \boldsymbol{\alpha} \quad \text{on } \Gamma \end{aligned}$$

$\mathbf{A}$ ... Magnetic vector potential

Weak Form:

Find $\mathbf{A}_h \in V_\alpha := \{\mathbf{A}_h \in \mathcal{V}_h : \mathbf{A}_h \times \mathbf{n} = \alpha_h \text{ on } \Gamma\}$, such that

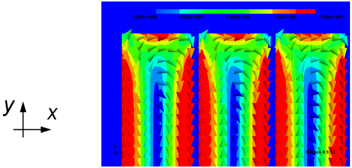
$$\int_\Omega \frac{1}{\mu} \operatorname{curl} \mathbf{A}_h \cdot \operatorname{curl} \mathbf{v}_h \, d\Omega + j\omega \int_\Omega \sigma \mathbf{A}_h \cdot \mathbf{v}_h \, d\Omega = \int_\Omega \mathbf{J} \cdot \mathbf{v}_h \, d\Omega$$

for all $\mathbf{v}_h \in V_0 := \{\mathbf{v}_h \in \mathcal{V}_h : \mathbf{v}_h \times \mathbf{n} = \mathbf{0} \text{ on } \Gamma\}$, where

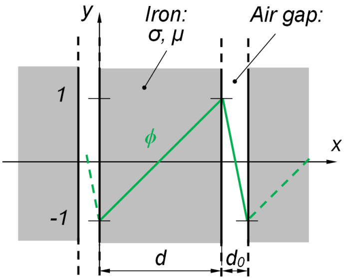
$\mathcal{V}_h \subset H(\operatorname{curl}, \Omega)$.

The solution serves as a reference solution for the multi-scale finite element methods.

Multi-Scale Approach:



Eddy currents in laminates, detail



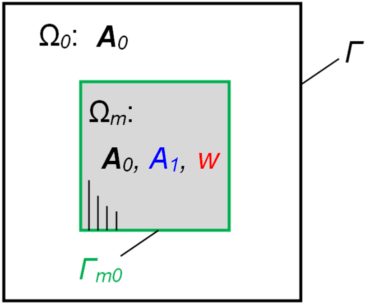
Micro-schape function for one periode $d + d_0$

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \nabla(\phi w)$$

Average value $\mathbf{A_0}$
Scalar quantities A_1 and w
Micro-shape function ϕ

Boundary Conditions:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \nabla(\phi w)$$



Outer boundary Γ:

$\mathbf{A}_0 \times \mathbf{n} = \boldsymbol{\alpha}$

Interface Γ_{m0}:

Natural boundary conditions

Weak Form of the MSFEM:

Inserting the multi-scale approach into the weak form yields

Find $(\mathbf{A}_{0h}, A_{1h}, w_h) \in V_B := \{(\mathbf{A}_{0h}, A_{1h}, w_h) : \boldsymbol{A}_{0h} \in \mathcal{U}_h, A_{1h} \in \mathcal{V}_h , w_h \in \mathcal{W}_h \text{ and } \boldsymbol{A}_{0h} \times \boldsymbol{n} = \alpha_h \text{ on } \Gamma_B\}$, such that

$$\int_\Omega \frac{1}{\mu} \operatorname{curl} \tilde{\mathbf{A}}_h \cdot \operatorname{curl} \tilde{\mathbf{v}}_h \, d\Omega + j\omega \int_\Omega \sigma \tilde{\mathbf{A}}_h \cdot \tilde{\mathbf{v}}_h \, d\Omega = \int_\Omega \mathbf{J} \cdot \tilde{\mathbf{v}}_h \, d\Omega$$

for all $(\mathbf{v}_{0h}, v_{1h}, q_h) \in V_0 := \{(\mathbf{v}_{0h}, v_{1h}, q_h) : \mathbf{v}_{0h} \in \mathcal{U}_h, v_{1h} \in \mathcal{V}_h, q_h \in \mathcal{W}_h \text{ and } \mathbf{v}_{0h} \times \mathbf{n} = \mathbf{0} \text{ on } \Gamma_B\}$.

Finite element subspaces:

$\mathbf{A}_{0h}, \;\; \mathbf{A}_{0h} \in \mathcal{U}_h \subset H(\operatorname{curl}, \Omega)$

$A_{1h}, \;\; v_{1h} \in \mathcal{V}_h \subset L_2(\Omega_m)$

$w_h, q_h \in \mathcal{W}_h \subset H^1(\Omega_m)$ and

$\phi \in H_{per}(\Omega_m)$

Finite Element System:

$$\int_\Omega \frac{1}{\mu} \operatorname{curl} \tilde{\mathbf{A}}_h \cdot \operatorname{curl} \tilde{\mathbf{v}}_h \, d\Omega + j\omega \int_\Omega \sigma \tilde{\mathbf{A}}_h \cdot \tilde{\mathbf{v}}_h \, d\Omega = \int_{\Omega_0} \mathbf{J}_0 \cdot \tilde{\mathbf{v}}_h \, d\Omega$$

$$\int_\Omega \frac{1}{\mu} \operatorname{curl} \left(\mathbf{A}_{0h} + \phi \begin{pmatrix} 0 \\ A_{1h} \end{pmatrix} + \nabla(\phi w_h) \right) \cdot \operatorname{curl} \left(\mathbf{v}_{0h} + \phi \begin{pmatrix} 0 \\ v_{1h} \end{pmatrix} + \nabla(\phi q_h) \right) \, d\Omega$$

$$+ j\omega \int_\Omega \sigma \left(\mathbf{A}_{0h} + \phi \begin{pmatrix} 0 \\ A_{1h} \end{pmatrix} + \nabla(\phi w_h) \right) \cdot \left(\mathbf{v}_{0h} + \phi \begin{pmatrix} 0 \\ v_{1h} \end{pmatrix} + \nabla(\phi q_h) \right) \, d\Omega =$$

$$\int_{\Omega_0} \mathbf{J_0} \cdot \left(\mathbf{v}_{0h} + \phi \begin{pmatrix} 0 \\ v_{1h} \end{pmatrix} + \nabla(\phi q_h) \right) \, d\Omega$$

Finite Element System:

$$\int_\Omega \frac{1}{\mu} \operatorname{curl}\left(\mathbf{A}_{0h} + \phi \begin{pmatrix} 0 \\ A_{1h} \end{pmatrix}\right) \cdot \operatorname{curl}\left(\mathbf{v}_{0h} + \phi \begin{pmatrix} 0 \\ v_{1h} \end{pmatrix}\right) \, d\Omega$$

$$+ j\omega \int_\Omega \sigma \left(\mathbf{A}_{0h} + \phi \begin{pmatrix} 0 \\ A_{1h} \end{pmatrix} + \nabla(\phi w_h)\right) \cdot \left(\mathbf{v}_{0h} + \phi \begin{pmatrix} 0 \\ v_{1h} \end{pmatrix} + \nabla(\phi q_h)\right) \, d\Omega =$$

$$\int_\Omega \begin{pmatrix} \operatorname{curl} \mathbf{A}_{0h} \\ A_1 \end{pmatrix}^T \begin{pmatrix} \nu & \nu\phi_x \\ \nu\phi_x & \nu\phi_x^2 \end{pmatrix} \begin{pmatrix} \operatorname{curl} \mathbf{v}_{0h} \\ v_1 \end{pmatrix} d\Omega \quad +$$

$$j\omega \int_\Omega \begin{pmatrix} (\mathbf{A}_{0h})_x \\ (\mathbf{A}_{0h})_y \\ A_{1h} \\ w_h \\ w_{hx} \\ w_{hy} \end{pmatrix}^T \begin{pmatrix} \sigma & 0 & 0 & \sigma\phi_x & \sigma\phi & 0 \\ 0 & \sigma & \sigma\phi & 0 & 0 & \sigma\phi \\ 0 & \sigma\phi & \sigma\phi^2 & 0 & 0 & \sigma\phi^2 \\ \sigma\phi_x & 0 & 0 & \sigma\phi_x^2 & \sigma\phi_x\phi & 0 \\ \sigma\phi & 0 & 0 & \sigma\phi_x\phi & \sigma\phi^2 & 0 \\ 0 & \sigma\phi & \sigma\phi^2 & 0 & 0 & \sigma\phi^2 \end{pmatrix} \begin{pmatrix} (\mathbf{v}_{0h})_x \\ (\mathbf{v}_{0h})_y \\ v_{1h} \\ q_h \\ q_{hx} \\ q_{hy} \end{pmatrix} d\Omega$$

with $\nu = \mu^{-1}$.

Homogenization:

Averaging of coefficients λ, $\lambda\nabla\phi$, $\lambda\phi$, etc. over the periode $p = d + d_0$ ($\phi_x := \nabla\phi$):

$$\begin{aligned}
\overline{\lambda} &= \frac{1}{p}\int_0^p \lambda(x)dx = \frac{\lambda_{Fe}d + \lambda_0 d_0}{p} \\
\overline{\lambda\phi_x} &= \frac{1}{p}\int_0^p \lambda(x)\phi_x(x)dx = 2\frac{\lambda_{Fe} - \lambda_0}{p} \\
\overline{\lambda\phi} &= \frac{1}{p}\int_0^p \lambda(x)\phi(x)dx = 0 \\
\overline{\lambda\phi_x^2} &= \frac{1}{p}\int_0^p \lambda(x)\phi_x(x)\phi_x(x)dx = \frac{4}{p}(\frac{\lambda_{Fe}}{d} + \frac{\lambda_0}{d_0}) \\
\overline{\lambda\phi_x\phi} &= \frac{1}{p}\int_0^p \lambda(x)\phi_x(x)\phi(x)dx = 0 \\
\overline{\lambda\phi^2} &= \frac{1}{p}\int_0^p \lambda(x)\phi(x)\phi(x)dx = \frac{\lambda_{Fe}d + \lambda_0 d_0}{3p}
\end{aligned}$$

Homogenized Finite Element System:

$$\int_\Omega \begin{pmatrix} \operatorname{curl} \overline{\mathbf{A}}_{0h} \\ \overline{A}_{1h} \end{pmatrix}^T \begin{pmatrix} \overline{\nu} & \overline{\nu\phi_x} \\ \overline{\nu\phi_x} & \overline{\nu\phi_x^2} \end{pmatrix} \begin{pmatrix} \operatorname{curl} \overline{\mathbf{v}}_{0h} \\ \overline{v}_{1h} \end{pmatrix} d\Omega, \quad +$$

$$j\omega \int_\Omega \begin{pmatrix} (\overline{\mathbf{A}}_{0h})_x \\ (\overline{\mathbf{A}}_{0h})_y \\ \overline{A}_{1h} \\ \overline{w}_h \\ \overline{w}_{hx} \\ \overline{w}_{hy} \end{pmatrix}^T \begin{pmatrix} \overline{\sigma} & 0 & 0 & \overline{\sigma\phi_x} & \overline{\sigma\phi} & 0 \\ 0 & \overline{\sigma} & \overline{\sigma\phi} & 0 & 0 & \overline{\sigma\phi} \\ 0 & \overline{\sigma\phi} & \overline{\sigma\phi^2} & 0 & 0 & \overline{\sigma\phi^2} \\ \overline{\sigma\phi_x} & 0 & 0 & \overline{\sigma\phi_x^2} & \overline{\sigma\phi_x\phi} & 0 \\ \overline{\sigma\phi} & 0 & 0 & \overline{\sigma\phi_x\phi} & \overline{\sigma\phi^2} & 0 \\ 0 & \overline{\sigma\phi} & \overline{\sigma\phi^2} & 0 & 0 & \overline{\sigma\phi^2} \end{pmatrix} \begin{pmatrix} (\overline{\mathbf{v}}_{0h})_x \\ (\overline{\mathbf{v}}_{0h})_y \\ \overline{v}_{1h} \\ \overline{q}_h \\ \overline{q}_{hx} \\ \overline{q}_{hy} \end{pmatrix} d\Omega$$

with $\nu = \mu^{-1}$.

Higher Order Multi-Scale Approach:

First order:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \nabla(\phi w)$$

Higher order:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_{12} \end{pmatrix} + \phi_3 \begin{pmatrix} 0 \\ A_{32} \end{pmatrix} + \phi_5 \begin{pmatrix} 0 \\ A_{52} \end{pmatrix}$$
$$+ \nabla(\phi_1 w_1) + \nabla(\phi_3 w_3) + \nabla(\phi_5 w_5)$$

Finite element subspaces:

$\mathbf{A}_{0h} \in \mathcal{U}_h \subset H(\text{curl}, \Omega)$

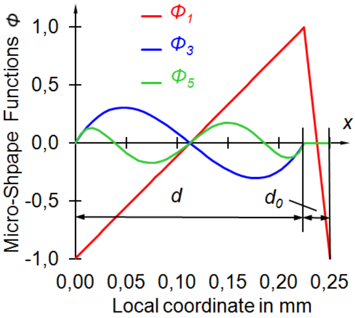
$A_{12h}, \; A_{32h}, \; A_{52h} \; \in \mathcal{V}_h \subset L_2(\Omega_m)$

$w_{1h}, \; w_{3h}, \; w_{5h} \; \in \mathcal{W}_h \subset H^1(\Omega_m)$ and

$\phi_1, \; \phi_3, \; \phi_5 \in H_{per}(\Omega_m)$

Higher Order Multi-Scale Approach:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_{12} \end{pmatrix} + \phi_3 \begin{pmatrix} 0 \\ A_{32} \end{pmatrix} + \phi_5 \begin{pmatrix} 0 \\ A_{52} \end{pmatrix} + \nabla(\phi_1 w_1) + \nabla(\phi_3 w_3) + \nabla(\phi_5 w_5)$$

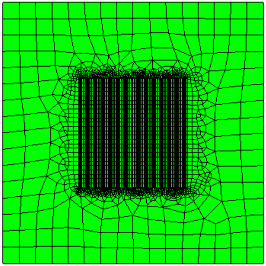


Higher order micro-shape functions:
Integrated Lengendre polynomials

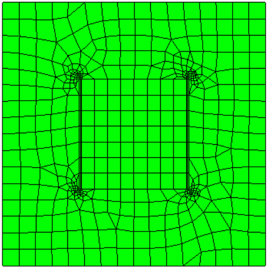
Numerical Example:

$$\sigma = 2 \cdot 10^6 S/m$$
$$ff = 0.9 \text{ (Fill factor)}$$
$$\alpha = 4 \cdot 10^{-3} Vs/m$$

$$\mu_r = 50,000$$
$$n = 100 \text{ No. Laminates}$$
$$d = d + d_0 = 0.25mm$$



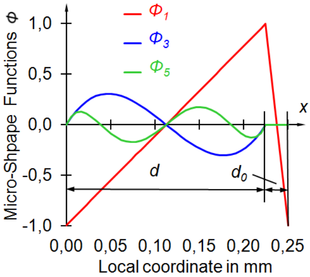
Referenz Model (RM):
Laminates are resolved



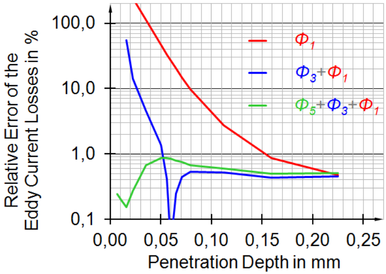
Homogenization Model (HM):
Laminates are not resolved

Higher Order Multi-Scale Approach:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_{12} \end{pmatrix} + \phi_3 \begin{pmatrix} 0 \\ A_{32} \end{pmatrix} + \phi_5 \begin{pmatrix} 0 \\ A_{52} \end{pmatrix} + \nabla(\phi_1 w_1) + \nabla(\phi_3 w_3) + \nabla(\phi_5 w_5)$$



Higher order micro-shape functions: Integrated Lengendre polynomials



Comparison of the eddy current losses in the frequency domain

Higher Order Multi-Scale Approach:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_{12} \end{pmatrix} + \phi_3 \begin{pmatrix} 0 \\ A_{32} \end{pmatrix} + \phi_5 \begin{pmatrix} 0 \\ A_{52} \end{pmatrix} + \nabla(\phi_1 w_1) + \nabla(\phi_3 w_3) + \nabla(\phi_5 w_5)$$

Numerical Data:

Table: Number of degrees of freedom for the most accurate approach.

	Total No.	$H(\text{curl}, \Omega)$	$L_2(\Omega_m)$	$L_2(\Omega_m)$	$L_2(\Omega_m)$	$H^1(\Omega_m)$
RS	123,564 [a])	123564	-	-		
MSFEM	1,194 [b])	630	317	317	317	314

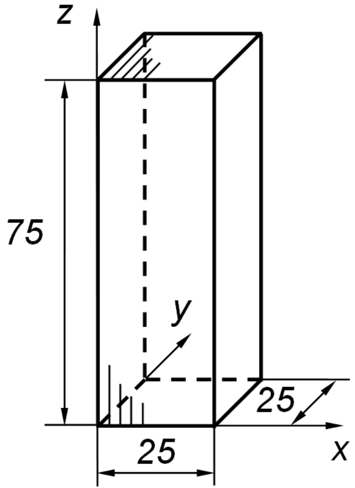
[a]) For 2^{nd} order $H(\text{curl})$ - elements.

[b]) Schur complement method eliminates all degrees of freedom of $L_2(\Omega)$ and possible higher order ones of $H(\text{curl}, \Omega)$ and $H^1(\Omega_m)$.

Neglecting $\nabla(\phi_3 w_3)$ and $\nabla(\phi_5 w_5)$ yields practically the same accuracy.

A similar problem as in Fig. 3 but with $1,000$ laminates uses about twice the amount of computer resources.

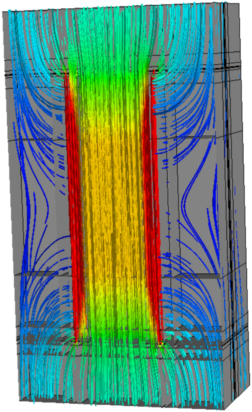
Model Problem:



$$\sigma = 2 \cdot 10^6 S/m$$
$$\mu_r = 50,000$$
$$|\alpha| = 6 \cdot 10^{-3} Vs/m$$
$$n = 100 \text{ No. Laminates}$$
$$d = d + d_0 = 0.25mm$$
$$ff = 0.9 \text{ (Fill factor)}$$

Problem with dimensions in mm.

Magnetic Field:



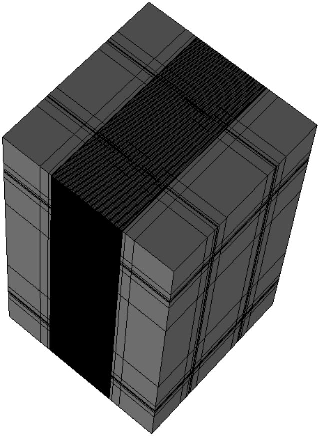
1.) Large main magnetic field

2.) Significant magnetic stray field

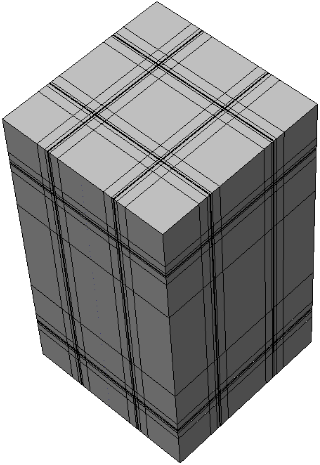
3.) Pronounced boundary layers

Challenges for the MSFEM in 3D.

Finite Element Models:



FE model for the reference solution (RS).



FE model for MSFEM.

Multi-Scale Approach for 3D:

2D:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \nabla(\phi w)$$

3D: (straightforward extension)

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + \nabla(\phi w)$$

Finite element subspaces:

$\mathbf{A}_{0h} \in \mathcal{U}_h \subset H(\mathrm{curl}, \Omega)$

$A_{12h}, A_{13h} \in \mathcal{V}_h \subset L_2(\Omega_m)$

$w_h \in \mathcal{W}_h \subset H^1(\Omega_m)$

and $\phi \in H_{per}(\Omega_m)$

Multi-Scale Approach for 3D:

3D: (straightforward extension)

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + \nabla(\phi w)$$

Table: Comparison of Eddy Current Losses

	RS	MSA
Losses in W	1.787	4.387
Rel. Error in %	-	145.0

Study of Different Multi-Scale Approaches (MSA):

$$MSA1) \qquad \tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + \nabla(\phi w), \quad w \in H^1(\Omega_m)$$

$$MSA2) \qquad \tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix}$$

$$MSA3) \qquad \tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + w \begin{pmatrix} \phi_x \\ 0 \\ 0 \end{pmatrix}, \quad w \in L_2(\Omega_m)$$

$$MSA4) \qquad \tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + w \begin{pmatrix} \phi_x \\ 0 \\ 0 \end{pmatrix}, \quad w \in H^1(\Omega_m)$$

Finite element subspaces:

$\mathbf{A}_{0h} \in \mathcal{U}_h \subset H(\mathrm{curl}, \Omega)$, $\quad A_{12h}, A_{13h} \in \mathcal{V}_h \subset L_2(\Omega_m)$ and $\phi \in H_{per}(\Omega_m)$

Study of Different Multi-Scale Approaches (MSA):

$$MSA1) \qquad \tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + \nabla(\phi w), \quad w \in H^1(\Omega_m)$$

Table: Comparison of Eddy Current Losses.

	RS	MSA1	MSA2	MSA3	MSA4
Losses in W	1.787	4.387	78.04	0.927	1.854
Rel. Error in %	-	145.0	4267	-48.13	3.75

Study of Different Multi-Scale Approaches (MSA):

$$MSA2) \qquad \tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix}$$

Table: Comparison of Eddy Current Losses.

	RS	MSA1	MSA2	MSA3	MSA4
Losses in W	1.787	4.387	78.04	0.927	1.854
Rel. Error in %	-	145.0	4267	-48.13	3.75

Study of Different Multi-Scale Approaches (MSA):

$$MSA3) \qquad \tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + w \begin{pmatrix} \phi_x \\ 0 \\ 0 \end{pmatrix}, \quad w \in L_2(\Omega_m)$$

Table: Comparison of Eddy Current Losses.

	RS	MSA1	MSA2	MSA3	MSA4
Losses in W	1.787	4.387	78.04	0.927	1.854
Rel. Error in %	-	145.0	4267	-48.13	3.75

Study of Different Multi-Scale Approaches (MSA):

$$MSA4) \qquad \tilde{\mathbf{A}} = \mathbf{A_0} + \phi \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + w \begin{pmatrix} \phi_x \\ 0 \\ 0 \end{pmatrix}, \quad w \in H^1(\Omega_m)$$

Table: Comparison of Eddy Current Losses.

	RS	MSA1	MSA2	MSA3	MSA4
Losses in W	1.787	4.387	78.04	0.927	1.854
Rel. Error in %	-	145.0	4267	-48.13	3.75

Study of Different Multi-Scale Approaches (MSA):

$$MSA1)\quad \operatorname{curl}\tilde{\mathbf{A}} = \operatorname{curl}\mathbf{A_0} + \phi_x \begin{pmatrix} 0 \\ -A_{13} \\ A_{12} \end{pmatrix} + \operatorname{curl}(\nabla(\phi w)), \quad w \in H^1(\Omega_m)$$

$$MSA2)\quad \operatorname{curl}\tilde{\mathbf{A}} = \operatorname{curl}\mathbf{A_0} + \phi_x \begin{pmatrix} 0 \\ -A_{13} \\ A_{12} \end{pmatrix}$$

$$MSA3)\quad \operatorname{curl}\tilde{\mathbf{A}} = \operatorname{curl}\mathbf{A_0} + \phi_x \begin{pmatrix} 0 \\ -A_{13} \\ A_{12} \end{pmatrix} + \operatorname{curl}\begin{pmatrix} w\phi_x \\ 0 \\ 0 \end{pmatrix}, \quad w \in L_2(\Omega_m)$$

$$MSA4)\quad \operatorname{curl}\tilde{\mathbf{A}} = \operatorname{curl}\mathbf{A_0} + \phi_x \begin{pmatrix} 0 \\ -A_{13} \\ A_{12} \end{pmatrix} + \phi_x \begin{pmatrix} 0 \\ w_z \\ -w_y \end{pmatrix}, \quad w \in H^1(\Omega_m)$$

Finite element subspaces:

$\mathbf{A}_{0h} \in \mathcal{U}_h \subset H(\operatorname{curl},\Omega), \quad A_{12h}, A_{13h} \in \mathcal{V}_h \subset L_2(\Omega_m)$ and $\phi \in H_{per}(\Omega_m)$

Higher Order Multi-Scale Approach in 3D:

Only odd terms are considered ...

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + \phi_3 \begin{pmatrix} 0 \\ A_{32} \\ A_{33} \end{pmatrix} + w_1 \begin{pmatrix} \phi_{1x} \\ 0 \\ 0 \end{pmatrix} + w_3 \begin{pmatrix} \phi_{3x} \\ 0 \\ 0 \end{pmatrix}$$

Finite element subspaces:

$\mathbf{A}_{0h} \in \mathcal{U}_h \subset H(\text{curl}, \Omega)$

$A_{12h},\ A_{13h},\ A_{32h},\ A_{33h}\ \in \mathcal{V}_h \subset L_2(\Omega_m)$

$w_{1h},\ w_{3h}\ \in \mathcal{W}_h \subset H^1(\Omega_m)$ and

$\phi_1,\ \phi_3 \in H_{per}(\Omega_m)$

Anisotropic Material:

Unit vectors:

$\mathbf{e}_\perp$ ... perpendicular to the lamination ($\mathbf{e}_\perp = \mathbf{e}_\mathbf{x}$)
$\mathbf{e}_\parallel$ ... any other vector ($\mathbf{e}_\parallel \cdot \mathbf{e}_\perp = 0$)

Conductivity:

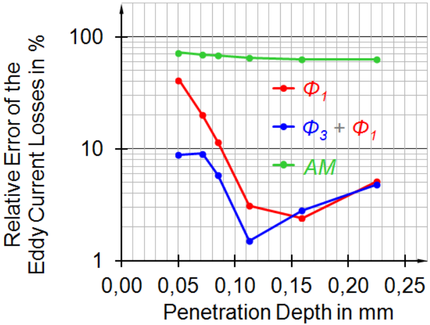
$$\sigma = \begin{pmatrix} \sigma_\perp & 0 & 0 \\ 0 & \sigma_\parallel & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix}, \quad \sigma_\perp = 0, \quad \sigma_\parallel = k_f \sigma_{Fe}$$

Reluctivity:

$$\nu = \begin{pmatrix} \nu_\perp & 0 & 0 \\ 0 & \nu_\parallel & 0 \\ 0 & 0 & \nu_\parallel \end{pmatrix}, \quad \nu_\perp = \frac{k_f}{\mu_{Fe}} + \frac{1-k_f}{\mu_0}, \quad \nu_\parallel = \frac{1}{\mu_\parallel} \approx \frac{1}{k_f \mu_{Fe}}$$

Higher Order Multi-Scale Approach in 3D:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + \phi_3 \begin{pmatrix} 0 \\ A_{32} \\ A_{33} \end{pmatrix} + w_1 \begin{pmatrix} \phi_{1x} \\ 0 \\ 0 \end{pmatrix} + w_3 \begin{pmatrix} \phi_{3x} \\ 0 \\ 0 \end{pmatrix}$$



Comparison of the eddy current losses in the frequency domain

Higher Order Multi-Scale Approach in 3D:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_{12} \\ A_{13} \end{pmatrix} + \phi_3 \begin{pmatrix} 0 \\ A_{32} \\ A_{33} \end{pmatrix} + w_1 \begin{pmatrix} \phi_{1x} \\ 0 \\ 0 \end{pmatrix} + w_3 \begin{pmatrix} \phi_{3x} \\ 0 \\ 0 \end{pmatrix}$$

Numerical Data:

Table: Number of degrees of freedom.

	Total No.	$H(\text{curl}, \Omega)$	$L_2(\Omega_m)$	$H^1(\Omega_m)$
RS	4 401, 247	4 401, 247	-	-
MSFEM	161, 444	97, 774	10, 062 [a])	11, 711 [b])

[a]) Four times.

[b]) Two times.

Thank you for your attention!