Multiscale and Harmonic Balance FEM for the Eddy Current Problem in Laminated Iron Cores

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Der Wissenschaftsfonds

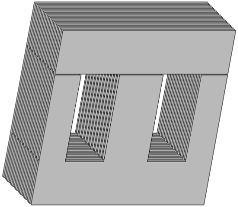
Outline

1. Introduction
2. Eddy Current Problem in 2D
3. Multiscale Finite Element Method MSFEM
4. Harmonic Balance Method HBM
5. Harmonic Balance Multiscale Finite Element Method HBMSFEM
6. Results

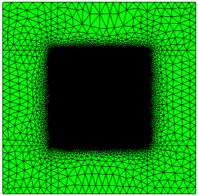
Introduction

Aim: Simulation of eddy current losses in large transformers

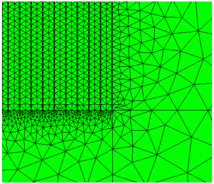
Large transformer



Laminated core

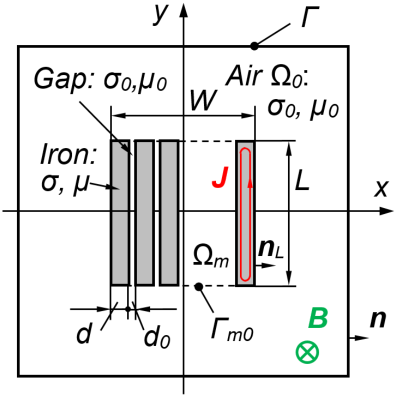


FE-Model with 100 laminates



Detail, lower right corner

Eddy Current Problem in 2D (Reference Solution):



Draft of the problem in 2D

Large scale dimensions:

W ... width
L ... length

Small scale dimensions:

$d + d_0$ ... iron + air
$k_f = \frac{d}{d+d_0}$ ... fill factor

Conductivity:

σ ... iron
σ_0 ... air

Permeability:

μ ... iron
μ_0 ... air

Quantities:

B ... Magnetic flux density
J ... Current density

Boundary Value Problem:

$$\begin{aligned} \operatorname{curl} \frac{1}{\mu} \operatorname{curl} \mathbf{A} + j\omega\sigma\mathbf{A} &= \mathbf{J}_0 \quad \text{in } \Omega = \Omega_m \cup \Omega_0, \\ \mathbf{A} \times \mathbf{n} &= \boldsymbol{\alpha} \quad \text{on } \Gamma \end{aligned}$$

$\mathbf{A}$ ... Magnetic vector potential

Weak Form:

Find $\mathbf{A}_h \in V_\alpha := \{\mathbf{A}_h \in \mathcal{V}_h : \mathbf{A}_h \times \mathbf{n} = \alpha_h \text{ on } \Gamma\}$, such that

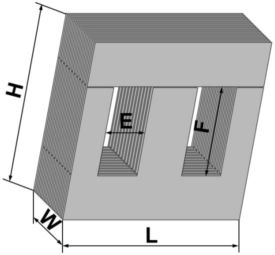
$$\int_\Omega \frac{1}{\mu} \operatorname{curl} \mathbf{A}_h \cdot \operatorname{curl} \mathbf{v}_h \, d\Omega + j\omega \int_\Omega \sigma \mathbf{A}_h \cdot \mathbf{v}_h \, d\Omega = \int_\Omega \mathbf{J}_0 \cdot \mathbf{v}_h \, d\Omega$$

for all $\mathbf{v}_h \in V_0 := \{\mathbf{v}_h \in \mathcal{V}_h : \mathbf{v}_h \times \mathbf{n} = \mathbf{0} \text{ on } \Gamma\}$, where

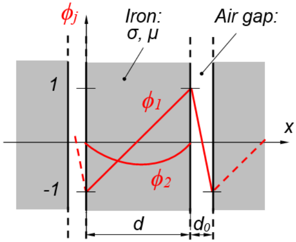
$\mathcal{V}_h \subset H(\operatorname{curl}, \Omega)$.

The solution serves as a reference solution for the harmonic balance multiscale finite element method (HBMSFEM).

Multiscale Finite Element Method MSFEM:



Large Scale



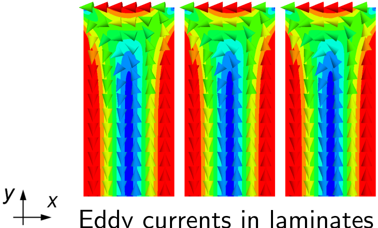
Microshape functions ϕ_i for one periode $d + d_0$

Multiscale finite element approach:

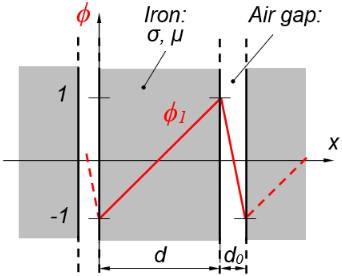
$$u_h(x) = \sum_i^n \sum_j^m u_{ij} \varphi_i(x) \phi_j(x) = \sum_i^n \sum_j^m u_{ij} \psi_{ij}(x)$$

Standard polynomials φ_i
Special functions, micro-shape functions ϕ_j
u_{ij} are the coefficients of the approximated solution u_h
New basis functions ψ_{ij}

Multiscale Approach:



Eddy currents in laminates with edge effect, detail



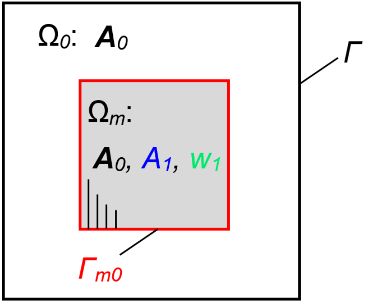
Micro-shape function for one periode $d + d_0$

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \nabla(\phi_1 w_1)$$

Average value $\mathbf{A_0}$
Scalar quantities A_1 and w_1
Micro-shape function ϕ_1

Boundary Conditions:

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1 \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \nabla(\phi_1 w_1)$$



Outer boundary Γ:

$$\mathbf{A}_0 \times \mathbf{n} = \mathbf{A}_t$$

Interface Γ_{m0}:

Natural boundary conditions

Weak Form of the MSFEM:

Inserting the multiscale approach into the weak form yields

Find $(\mathbf{A}_{0h}, A_{1h}, w_{1h}) \in V_B := \{(\mathbf{A}_{0h}, A_{1h}, w_{1h}) : \boldsymbol{A}_{0h} \in \mathcal{U}_h, A_{1h} \in \mathcal{V}_h , w_{1h} \in \mathcal{W}_h$ and $\boldsymbol{A}_{0h} \times \boldsymbol{n} = \alpha_h$ on $\Gamma\}$, such that

$$\int_\Omega \frac{1}{\mu} \operatorname{curl} \tilde{\mathbf{A}}_h \cdot \operatorname{curl} \tilde{\mathbf{v}}_h \, d\Omega + j\omega \int_\Omega \sigma \tilde{\mathbf{A}}_h \cdot \tilde{\mathbf{v}}_h \, d\Omega = \int_\Omega \mathbf{J}_0 \cdot \tilde{\mathbf{v}}_h \, d\Omega$$

for all $(\mathbf{v}_{0h}, v_{1h}, q_{1h}) \in V_0$.

Finite element subspaces:

$\mathbf{A}_{0h},\ \mathbf{v}_{0h} \in \mathcal{U}_h \subset H(\operatorname{curl}, \Omega)$

$A_{1h},\ v_{1h} \in \mathcal{V}_h \subset L_2(\Omega_m)$

$w_{1h}, q_{1h} \in \mathcal{W}_h \subset H^1(\Omega_m)$ and

$\phi_1 \in H_{per}(\Omega_m)$

Harmonic Balance Method (HBM):

- Time harmonic excitation
- Nonlinear problem
- Steady state solution is only of interest

Steady state solution $u(\mathbf{x}, t)$ is periodic in time with period T:

$$u(\mathbf{x}, t) = u(\mathbf{x}, t + T), \qquad t \in \mathbb{R}$$

Truncated Fourier series

$$u(\mathbf{x}) = u_0(\mathbf{x}) + \sum_{k=1}^{N} u^c_{2k-1}(\mathbf{x}) \cos((2k-1)\omega t) + u^s_{2k-1}(\mathbf{x}) \sin((2k-1)\omega t)$$

with superscripts

$$\begin{cases} c \ldots \text{cosine} \\ s \ldots \text{sine} \end{cases}$$ and with the upper bound N.

Harmonic Balance Multiscale FEM (HBMSFEM):

Approach

$$\hat{\mathbf{A}} = \mathbf{A}_0(\mathbf{x}) + \sum_{k=1}^{N} \widetilde{\mathbf{A}}^{c}_{2k-1}(\mathbf{x}) \cos((2k-1)\omega t) + \widetilde{\mathbf{A}}^{s}_{2k-1}(\mathbf{x}) \sin((2k-1)\omega t)$$

with coefficient functions

$$\widetilde{\mathbf{A}}^{\alpha}_{2k-1}(\mathbf{x}) = \mathbf{A}^{\alpha}_{0,2k-1}(\mathbf{x}) + \phi_1 \begin{pmatrix} 0 \\ A^{\alpha}_{1,2k-1}(\mathbf{x}) \end{pmatrix} + \nabla \left(\phi_1 w^{\alpha}_{1,2k-1}(\mathbf{x})\right),$$

where $\alpha = c, s$ and $k \in \mathbb{N}$, $k \leq N$.

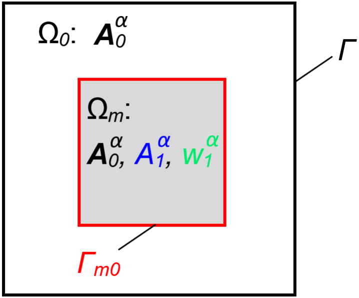
Integration over time

$$\int_{t=0}^{T} \int_{\Omega} \mu^{-1}(\hat{\mathbf{A}})\, \mathrm{curl}(\hat{\mathbf{A}}) \cdot \mathrm{curl}(\hat{\mathbf{v}})\, \mathrm{d}\Omega \mathrm{d}t + \int_{t=0}^{T} \int_{\Omega} \sigma \frac{\partial \hat{\mathbf{A}}}{\partial t} \cdot \hat{\mathbf{v}}\, \mathrm{d}\Omega \mathrm{d}t = \int_{t=0}^{T} \int_{\Omega} \mathbf{J}_0 \cdot \hat{\mathbf{v}}\, \mathrm{d}\Omega \mathrm{d}t$$

is required for the weak form.

Boundary Conditions:

$$\tilde{\mathbf{A}} = \mathbf{A_0}^{\alpha} + \phi_1 \begin{pmatrix} 0 \\ A_1^{\alpha} \end{pmatrix} + \nabla(\phi_1 w_1^{\alpha}), \qquad \alpha = c, s$$



Outer boundary Γ:

$$\mathbf{A}_0^{\alpha} \times \mathbf{n} = \mathbf{A}_t$$

Interface Γ_{m0}:

Natural boundary conditions

Weak Form of the HBMSFEM:

Find $(\mathbf{A}^{\alpha}_{0h,\beta},\ A^{\alpha}_{1h,\beta},\ w^{\alpha}_{1h,\beta}) \in V_{\boldsymbol{\alpha}_h} := \{(\mathbf{A}^{\alpha}_{0h,\beta},\ A^{\alpha}_{1h,\beta},\ w^{\alpha}_{1h,\beta}) : \mathbf{A}^{\alpha}_{0h,\beta} \in \mathcal{U}_h,\ A^{\alpha}_{1h,\beta} \in \mathcal{V}_h\ ,\ w^{\alpha}_{1h,\beta} \in \mathcal{W}_h$ and $\mathbf{A}^{\alpha}_{0h,\beta} \times \mathbf{n} = \boldsymbol{\alpha}_h$ on $\Gamma\}$, such that

$$\int_{t=0}^{T} \int_{\Omega} \mu^{-1}(\hat{\mathbf{A}})\,\mathrm{curl}(\hat{\mathbf{A}})\cdot\mathrm{curl}(\hat{\mathbf{v}})\,\mathrm{d}\Omega\mathrm{d}t + \int_{t=0}^{T} \int_{\Omega} \sigma \frac{\partial \hat{\mathbf{A}}}{\partial t}\cdot\hat{\mathbf{v}}\,\mathrm{d}\Omega\mathrm{d}t = \int_{t=0}^{T} \int_{\Omega} \mathbf{J}_0\cdot\hat{\mathbf{v}}\,\mathrm{d}\Omega\mathrm{d}t$$

for all $(\mathbf{v}^{\alpha}_{0h,\beta},\ v^{\alpha}_{1h,\beta},\ q^{\alpha}_{1h,\beta}) \in V_0$, where $\alpha = c, s$ and $\beta = 2k-1$, $k \in \mathbb{N} \quad \wedge \quad k \leq N$.

Finite element subspaces:

$$\mathbf{A}^{\alpha}_{0h,\beta},\ \mathbf{v}^{\alpha}_{0h,\beta} \in \mathcal{U}_h \subset H(\mathrm{curl},\Omega),$$
$$A^{\alpha}_{1h,\beta},\ v^{\alpha}_{1h,\beta} \in \mathcal{V}_h \subset L_2(\Omega_m),$$
$$w^{\alpha}_{1h,\beta}, q^{\alpha}_{1h,\beta} \in \mathcal{W}_h \subset H^1(\Omega_m) \text{ and}$$
$$\phi_1 \in H_{per}(\Omega_m)$$

Properties of Fourier Expansion:

Mass term

$$\int_{t=0}^{T} \int_{\Omega} \sigma \frac{\partial \hat{\mathbf{A}}}{\partial t} \cdot \hat{\mathbf{v}} \,\mathrm{d}\Omega \mathrm{d}t$$

a) Mass matrix is skew symmetric:

$$\frac{\partial}{\partial t} \left(u_{2k-1}^{c}(\mathbf{x}) \cdot \cos\left((2k-1)\omega t\right) + u_{2k-1}^{s}(\mathbf{x}) \sin\left((2k-1)\omega t\right) \right)$$
$$= (2k-1)\omega \left(-u_{2k-1}^{c}(\mathbf{x}) \sin\left((2k-1)\omega t\right) + u_{2k-1}^{s}(\mathbf{x}) \cos\left((2k-1)\omega t\right) \right)$$

b) Mass matrix is sparse:

Orthogonality of trigonometric functions:

$$\int_{t=0}^{T} \cos(i\omega t) \sin(j\omega t) \,\mathrm{d}t = 0,$$

$$\int_{t=0}^{T} \cos(i\omega t) \cos(j\omega t) \,\mathrm{d}t = \begin{cases} \frac{T}{2} \ldots i = j \\ 0 \ldots i \neq j, \end{cases}$$

$$\int_{t=0}^{T} \sin(i\omega t) \sin(j\omega t) \,\mathrm{d}t = \begin{cases} \frac{T}{2} \ldots i = j \\ 0 \ldots i \neq j \end{cases} \quad i, j \in \mathbb{N}$$

c) Since σ is constant, integration over time can be done analytically
d) Harmonics are decoupled

Stiffness term:

$$\int_{t=0}^{T} \int_{\Omega} \mu^{-1}(\hat{\mathbf{A}}) \, \mathrm{curl}\, \hat{\mathbf{A}} \cdot \mathrm{curl}\, \hat{\mathbf{v}} \, \mathrm{d}\Omega \mathrm{d}t$$

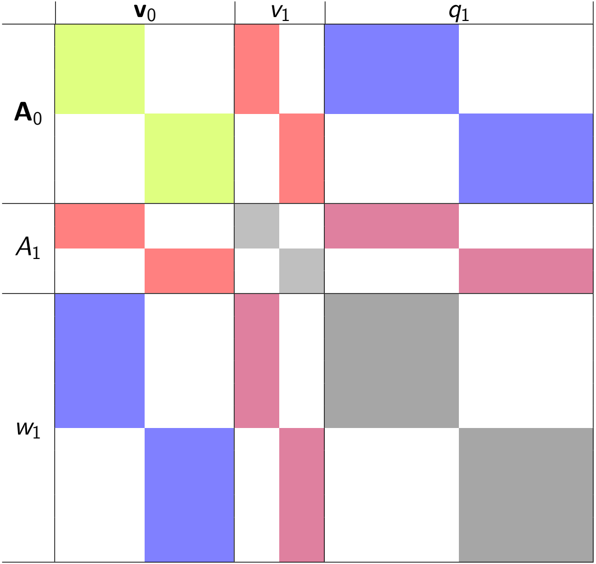
a) μ linear:

- Analytical integration over time
- Matrix is symmetric
- Harmonics are decoupled

b) $\mu(\hat{\mathbf{A}})$ nonlinear:

- Numerical integration over time is required
- Period T is subdivided uniformly into N_T sub-intervals
- Matrix is symmetric
- Harmonics are coupled

Mass matrix:



Pattern due to specific arrangement of FE-spaces.

Mass matrix (detail):

	$(\mathbf{v}^c_{0,1})_x$	$(\mathbf{v}^c_{0,1})_y$	$(\mathbf{v}^s_{0,1})_x$	$(\mathbf{v}^s_{0,1})_y$	$(\mathbf{v}^c_{0,3})_x$	$(\mathbf{v}^c_{0,3})_y$	$(\mathbf{v}^s_{0,3})_x$	$(\mathbf{v}^s_{0,3})_y$
$(\mathbf{A}^c_{0,1})_x$			$-\pi\sigma$					
$(\mathbf{A}^c_{0,1})_y$				$-\pi\sigma$				
$(\mathbf{A}^s_{0,1})_x$	$\pi\sigma$							
$(\mathbf{A}^s_{0,1})_y$		$\pi\sigma$						
$(\mathbf{A}^c_{0,3})_x$							$-3\pi\sigma$	
$(\mathbf{A}^c_{0,3})_y$								$-3\pi\sigma$
$(\mathbf{A}^s_{0,3})_x$					$3\pi\sigma$			
$(\mathbf{A}^s_{0,3})_y$						$3\pi\sigma$		

skew symmetric

Mass matrix (detail):

	$v^c_{1,1}$	$v^s_{1,1}$	$v^c_{1,1}$	$v^s_{1,1}$
$(\mathbf{A}^c_{0,1})_x$				
$(\mathbf{A}^c_{0,1})_y$		$-\phi_1\pi\sigma$		
$(\mathbf{A}^s_{0,1})_x$				
$(\mathbf{A}^s_{0,1})_y$	$\phi_1\pi\sigma$			
$(\mathbf{A}^c_{0,3})_x$				
$(\mathbf{A}^c_{0,3})_y$				$-3\phi_1\pi\sigma$
$(\mathbf{A}^s_{0,3})_x$				
$(\mathbf{A}^s_{0,3})_y$			$3\phi_1\pi\sigma$	

	$(\mathbf{v}^c_{0,1})_x$	$(\mathbf{v}^c_{0,1})_y$	$(\mathbf{v}^s_{0,1})_x$	$(\mathbf{v}^s_{0,1})_y$	$(\mathbf{v}^c_{0,3})_x$	$(\mathbf{v}^c_{0,3})_y$	$(\mathbf{v}^s_{0,3})_x$	$(\mathbf{v}^s_{0,3})_y$
$A^c_{1,1}$				$-\phi_1\pi\sigma$				
$A^s_{1,1}$		$\phi_1\pi\sigma$						
$A^c_{1,3}$								$-3\phi_1\pi\sigma$
$A^s_{1,3}$						$3\phi_1\pi\sigma$		

skew symmetric

Stiffness matrix (linear):

	curl $\mathbf{v}_{0,1}^c$	curl $\mathbf{v}_{0,1}^s$	curl $\mathbf{v}_{0,3}^c$	curl $\mathbf{v}_{0,3}^s$	$v_{1,1}^c$	$v_{1,1}^s$	$v_{1,3}^c$	$v_{1,3}^s$
curl $\mathbf{A}_{0,1}^c$	$\nu T/2$							
curl $\mathbf{A}_{0,1}^s$		$\nu T/2$						
curl $\mathbf{A}_{0,3}^c$			$\nu T/2$				symmetric	
curl $\mathbf{A}_{0,3}^s$				$\nu T/2$				
$A_{1,1}^c$	$\phi_{1,x}\nu T/2$				$\phi_{1,x}\phi_{1,x}\nu T/2$			
$A_{1,1}^s$		$\phi_{1,x}\nu T/2$				$\phi_{1,x}\phi_{1,x}\nu T/2$		
$A_{1,3}^c$			$\phi_{1,x}\nu T/2$				$\phi_{1,x}\phi_{1,x}\nu T/2$	
$A_{1,3}^s$				$\phi_{1,x}\nu T/2$				$\phi_{1,x}\phi_{1,x}\nu T/2$

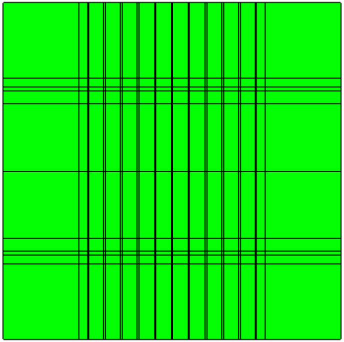
$$\nu = \mu^{-1}, \quad \phi_{1,x} = \partial_x \phi_1$$

Stiffness matrix (nonlinear):

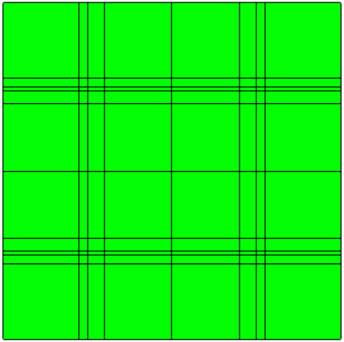
	curl $\mathbf{v}_{0,1}^c$	curl $\mathbf{v}_{0,1}^s$	curl $\mathbf{v}_{0,3}^c$	curl $\mathbf{v}_{0,3}^s$	$v_{1,1}^c$	$v_{1,1}^s$	$v_{1,3}^c$	$v_{1,3}^s$
curl $\mathbf{A}_{0,1}^c$	$\nu c_1 c_1$							
curl $\mathbf{A}_{0,1}^s$	$\nu s_1 c_1$	$\nu s_1 s_1$						
curl $\mathbf{A}_{0,3}^c$	$\nu c_3 c_1$	$\nu c_3 s_1$	$\nu c_3 c_3$				symmetric	
curl $\mathbf{A}_{0,3}^s$	$\nu s_3 c_1$	$\nu s_3 s_1$	$\nu s_3 c_3$	$\nu s_3 s_3$				
$A_{1,1}^c$	$\phi_{1,x}\nu c_1 c_1$	$\phi_{1,x}\nu c_1 s_1$	$\phi_{1,x}\nu c_1 c_3$	$\phi_{1,x}\nu c_1 s_3$	$\phi_{1,x}\phi_{1,x}\nu c_1 c_1$			
$A_{1,1}^s$	$\phi_{1,x}\nu s_1 c_1$	$\phi_{1,x}\nu s_1 s_1$	$\phi_{1,x}\nu s_1 c_3$	$\phi_{1,x}\nu s_1 s_3$	$\phi_{1,x}\phi_{1,x}\nu s_1 c_1$	$\phi_{1,x}\phi_{1,x}\nu s_1 s_1$		
$A_{1,3}^c$	$\phi_{1,x}\nu c_3 c_1$	$\phi_{1,x}\nu c_3 s_1$	$\phi_{1,x}\nu c_3 c_3$	$\phi_{1,x}\nu c_3 s_1$	$\phi_{1,x}\phi_{1,x}\nu c_3 c_1$	$\phi_{1,x}\phi_{1,x}\nu c_3 s_1$	$\phi_{1,x}\phi_{1,x}\nu c_3 c_3$	
$A_{1,3}^s$	$\phi_{1,x}\nu s_3 c_1$	$\phi_{1,x}\nu s_3 s_1$	$\phi_{1,x}\nu s_3 c_3$	$\phi_{1,x}\nu s_3 s_3$	$\phi_{1,x}\phi_{1,x}\nu s_3 c_1$	$\phi_{1,x}\phi_{1,x}\nu s_3 s_1$	$\phi_{1,x}\phi_{1,x}\nu s_3 c_3$	$\phi_{1,x}\phi_{1,x}\nu s_3 s_3$

$\nu = \nu(\hat{\mathbf{A}}) = \mu^{-1}(\hat{\mathbf{A}}), \quad \phi_{1,x} = \partial_x \phi_1,$ is yet to be integrated over the time

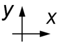
Finite Element Models:



FE model for the reference solution (RS).

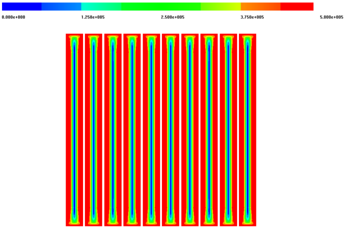


FE model for the MSFEM.

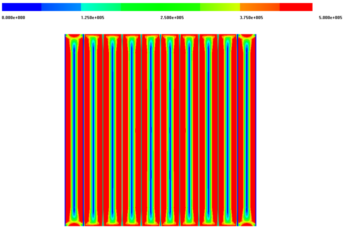


Discretisation in *y*-direction is the same for both FE models.

Magnitude of current density:

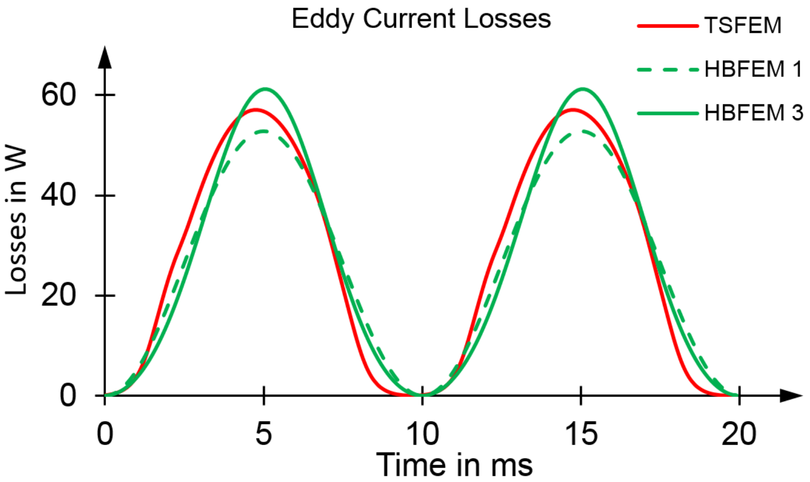


HBFEM



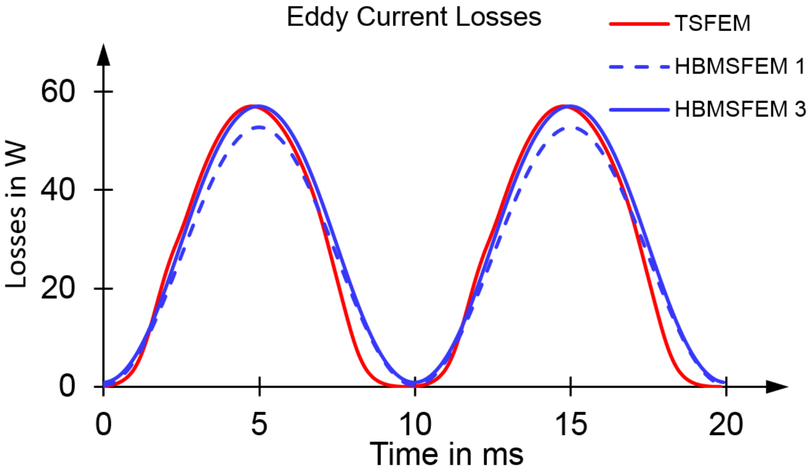
HBMSFEM

Instantaneous Losses:



HBFEM with fundamental waves only (1) and additionally with 3^{rd} order harmonics (3)

Instantaneous Losses:



HBMSFEM with fundamental waves only (1) and additionally with 3^{rd} order harmonics (3)

Thank you for your attention!