Nonasymptotic Homogenization of Laminated Magnetic Cores

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We are developing a rigorous framework for homogenization of laminated magnetic cores in the non-asymptotic case, when the spatial period of the structure is not vanishingly small relative to the penetration depth. The asymptotic limit not being applicable, the effective tensor may depend on the geometry of the core. Our model is valid for a realistic cylindrical geometry typical for rotating electrical machines, and for any reasonable size and composition of the lamination lattice cell. In this paper, the model is applied in the frequency domain under the assumption of linearity, but extensions to nonlinearity and hysteresis are discussed.

***Index Terms*—Homogenization, laminated cores, multiscale, eddy currents.**

I. Introduction and Formulation

HOmogenization of laminated cores is an old and important problem which has been extensively studied but not yet completely solved. To eliminate eddy currents in the bulk, thin insulation-coated laminations are typically used in various electrical machines. Eddy currents are then confined to the individual magnetic sheets, thereby reducing the losses. To make analysis and computer simulation feasible in practice, the fine structure of the laminations has to be "homogenized" – i.e. replaced with an approximately equivalent homogeneous material of the same size and shape.

The setup is schematically shown in Fig. 1 and can be summarized as follows. One lamination is represented in the cylindrical coordinates (r, z, θ) as a rectangle $\Omega = [r_{\text{in}}, r_{\text{out}}] \times [-a/2, a/2]$, where a is the lattice size (spatial period) in the z direction; the $\cos p\theta$ or $\sin p\theta$ variation of various field components is assumed in the angular direction (p being the number of pole pairs). Periodic boundary conditions link the fields at $z = \pm a/2$; Dirichlet conditions approximate the field on the rotor side of the airgap ($r = r_{\text{in}}$). Dirichlet-to-Neumann conditions on the outer side of the core ($r = r_{\text{in}}$) represent the field in the semi-infinite air strip. The magnetic field $\mathbf{h}(\mathbf{r}, t)$ in the original fine structure, and the respective "macroscale" field in an equivalent homogeneous medium satisfy the standard eddy current equations, with the displacement current neglected:

$$\nabla \times \mathbf{e} = -\partial_t \mathbf{b}, \quad \nabla \times \mathbf{h} = \mathbf{j}, \quad \nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad \nabla \times \mathbf{H} = \mathbf{J} \qquad (1)$$

under the standard notation for frequency, magnetic permeability, and conductivity. Our focus is on the material relations:

$$\mathbf{j} = \sigma \mathbf{e}, \quad \mathbf{b} = \mathbf{b}(\mathbf{h}); \quad (\mathbf{B}, \mathbf{J})^T = \mathcal{M}(\mathbf{H}, \mathbf{E})^T \qquad (2)$$

Here $\mathcal{M}$ is the effective material tensor. While the fine-scale **b**-**h** relation can be nonlinear, the respective coarse-scale relation is assumed to be linearized, because otherwise the homogenized model may become too complicated to be attractive in practice. In partial compensation for that, the $\mathcal{M}$ tensor may in general include the **B**-**E** and **J**-**H** coupling terms as additional (albeit nonstandard) degrees of freedom.

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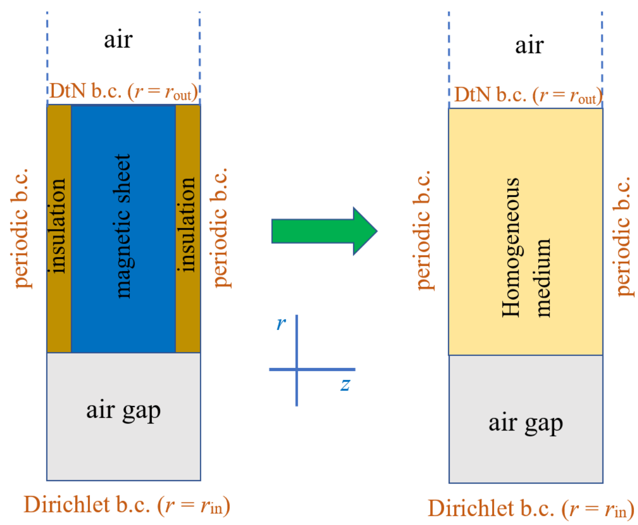

Fig. 1. Setup: homogenization problem for laminated structures in cylindrical geometry.

The crux of homogenization is the optimization problem, whose generic form is

$$\mathcal{M} = \operatorname{argmin}_{\mathcal{M}} \|F(\mathbf{H}, \mathbf{J}, \mathcal{M}) - f(\mathbf{h}, \mathbf{j})\| \qquad (3)$$

where F and f are the chosen goal functions and $\|\cdot\|$ is an appropriate norm. In our previous work on wave problems, the goal functions represented the far-field pattern of waves scattered from a periodic structure [10]–[12]. In that case, optimization (3) was facilitated by the availability of Bloch wave bases on the fine scale and their plane wave counterparts on the coarse scale. A semi-analytical procedure can therefore be constructed. This route is, unfortunately, not available for the eddy current problem under consideration. However, (3) can be solved numerically, since the size of the material tensor is relatively small, and since the absolute global minimum need not necessarily be guaranteed.

Clearly, the optimal effective tensor depends, in general, on the goal functions in (3). In wave problems, this manifests itself in the "uncertainty principle" [12]: non-asymptotic homogenization cannot in general predict, simultaneously and accurately, reaction fields *and* losses. With this in mind, for the lamination problem we explore goal functions containing both fields and losses with adjustable weights.

II. Proposed Approach vs. Existing Methods

A large number of semi-analytical [2], [3] and numerical (multiscale) models [1], [4]–[9] have been put forward, although no hard demarcation line between these types of models exists. In this paper, we draw on our experience in non-asymptotic homogenization for wave problems [10]–[12] to develop a homogenization procedure for laminated structures.

In the asymptotic case, when the cell size a is much smaller than the penetration depth in the conducting sheet, classic results on effective tensors of conductivity and permeability apply. In practice, however, the penetration depth is often comparable with the lamination thickness, and therefore a non-asymptotic analysis is called for. This has been widely recognized in the literature (see references above). Our non-asymptotic model is intended to include *as few simplification assumptions as possible*.

- Cylindrical geometry, typical for rotating electrical machines, is considered. In particular, a nontrivial field transition layer near the air gap is accurately represented.
- For asymptotic homogenization, the effective tensors are simple and virtually independent of the geometric features. This is not necessarily so in the non-asymptotic case, where these tensors may depend on the geometric dimensions.
- The homogenization procedure always involves only one lattice cell (one lamination), so the computational size of the problem is always small.

III. Numerical Example

As an illustrative example, the method was tested for the geometry shown in Fig. 1 using a relative magnetic permeability of 1000 and an electric conductivity $\sigma = 2.08 \times 10^6 S/m$. The iron fill factor is chosen as 0.95; the angular harmonic corresponds to $p = 1$. For a wide range of frequencies an optimized material tensor was constructed taking into account both the losses and the reaction field in the air gap. The results are visualized in Fig. 2.

As expected, for low frequencies the tensor obtained by classical homogenization is sufficient to give a good estimate of the total losses. The optimized tensor yields near perfect results even at high frequencies, where the assumptions of asymptotic homogenization do not hold anymore.

IV. Conclusion

We are developing numerical models for non-asymptotic homogenization of laminated magnetic cores. The approach borrows ideas from our non-asymptotic homogenization procedure in the wave case [10], [11]. Homogenization is applied to a realistic cylindrical geometry of the core, typical to rotating electrical machines. The model is valid for any reasonable size and composition of the lamination lattice cell. It has been shown to give uniformly higher accuracy than the standard homogenization method and by varying the optimization procedure it should be extendable to a far greater range of applications. The overriding objective is to develop a practical homogenization methodology relying on as few simplification assumptions as possible.

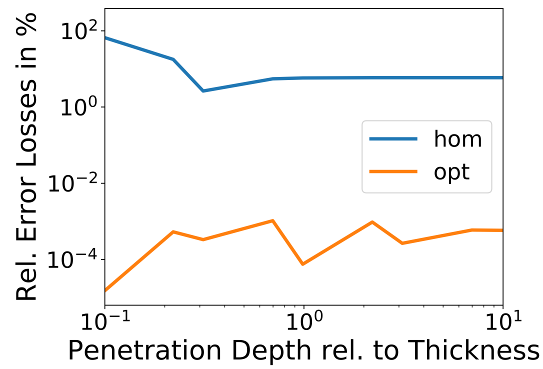

Fig. 2. The relative errors in the calculated losses for the classical homogenization (hom.) and the developed optimized tensor (opt.), depending on the relative penetration depth.

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