A 2D/1D Multiscale Finite Element Method Using the Biot-Savart Field for Electrical Machines

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A 2D/1D method for the A-formulation of the eddy current problem in a thin iron sheet is presented. It allows for the reduction of a three dimensional problem to a two dimensional one, where the coupling to the remaining one dimensional problem is already integrated implicitly into the two dimensional system. Similar to the classical multiscale finite element method this is achieved by separating the reference solution into what can be resolved on the two dimensional mesh and the rest which is resolved by an expansion using predefined polynomial shape functions. The presented method utilizes a modification to the previously presented version, which allows for the correct treatment of edge effects. Important novelties are the extension of the previously linear method to nonlinear materials and the application of systems which are driven by a prescribed Biot-Savart field. The proposed method is tested numerically in the context of a synchronous machine.

***Index Terms*—2D/1D problem, Biot-Savart field, eddy current problem, multiscale finite element method, nonlinear materials**

I. Introduction

THE SIMULATION of electrical machines is still an ongoing topic with many unsolved problems. Using the reasonable assumptions that all iron sheets are exposed to the same field, it is possible to restrict the simulation to a single sheet. However, solving the nonlinear eddy current problem on a single stator and rotor sheet using the three dimensional finite element method (FEM) is still a computationally expensive task [1].

A possible approach to further reduce the computational costs is to reduce the dimensionality of the problem using the fact that in a synchronous machine the thickness of an iron sheet is very small compared to its other dimensions. The early implementations of the 2D/1D idea iteratively solved a two dimensional static problem on the cross section and a one dimensional eddy current problem across the sheet thickness [2]. In more recent years variations have been presented, where the explicit coupling between the problems is avoided by the use of explicit expansions of the solution of the one dimensional problem, which is then included in the two dimensional problem directly [3]–[5].

A common shortcoming of these methods is the inability to correctly resolve the edge effects at the boundaries of the iron sheets. In [6] a linear method was presented which allowed for correctly resolved edge effects for the T-formulation. In this paper a modification is used, which includes the edge effects also for the A-formulation [7], and applied to a setting driven by a Biot-Savart field. Furthermore a way to extend the method to a nonlinear setting is discussed.

Numerical tests are carried out by comparison of the 2D/1D solution with the three dimensional reference solution of the eddy current problem on a synchronous machine.

II. The Eddy Current Problem

The domain Ω for the three dimensional reference solution consists of one stator sheet and one rotor sheet of a synchronous machine, see Fig. 1. It will be advantageous to use the decomposition $\Omega = \Omega_{2D} \times [-\frac{d}{2}, \frac{d}{2}]$ with the thickness d of the sheet, including the isolating layer, and the cross section of Ω at $z = 0$ denoted by Ω_{2D}.

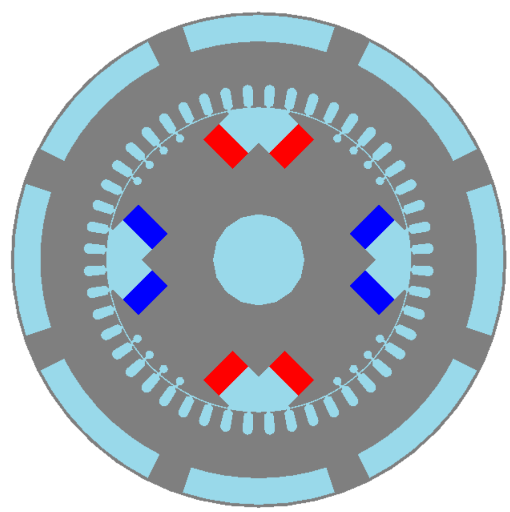

Fig. 1. The cross section of the machine Ω_{2D}. The materials are iron (grey) and air (light blue). The coils are visualized in red and blue, indicating a positive (red) or negative (blue) d.c. current density.

The problem is exited by d.c. currents in rotating coils. The eddy current problem in its weak formulation is given by: Find the magnetic vector potential $\mathbf{A} \in H(\mathrm{curl}, \Omega)$ fulfilling homogenous Dirichlet boundary conditions, so that for all $\mathbf{v} \in H(\mathrm{curl}, \Omega)$:

$$\int_\Omega \mu^{-1}(\mathrm{curl}\mathbf{A})\mathrm{curl}\mathbf{A}\cdot\mathrm{curl}\mathbf{v}+\sigma\frac{d\mathbf{A}}{dt}\cdot\mathbf{v} = \int_\Omega \mathbf{H}_{BS}\cdot\mathrm{curl}\mathbf{v}, \quad (1)$$

where μ denotes the magnetic permeability which depends on the flux density, σ the electric conductivity and $\mathbf{H}_{BS}$ the Biot-Savart field generated by the coils.

III. The 2D/1D Method

The main principle of the proposed method is to approximate the potential $\mathbf{A}$ by the multiscale approach

$$\mathbf{A} = \begin{pmatrix} \phi_1^0 \nabla u \\ 0 \end{pmatrix} + \begin{pmatrix} \phi_1 \mathbf{A}_1 \\ 0 \end{pmatrix} + \nabla(\phi_1 w_1) \tag{2}$$

with $u(x,y), w_1(x,y) \in H^1(\Omega_{2D})$ and $\mathbf{A}_1 \in H(\text{curl}, \Omega_{2D})$ as the new unknown functions and the predefined piecewise linear shape functions $\phi_0^1(z), \phi_1(z) \in H^1_{per}([-\frac{d}{2}, \frac{d}{2}])$.

The approach given in (2) allows for a good approximation of the potential $\mathbf{A}$, including the edge effects at the boundaries of the iron sheet. This is an improvement over the formula given in [6], where the edge effects cannot be resolved correctly for the A-formulation.

The approximation (2) is used in (1) for both the trial function $\mathbf{A}$ and the test function $\mathbf{v}$. Because the new unknown functions depend only on the coordinates of the cross section Ω_{2D}, the integration in z can be carried out analytically in the linear case (i.e. for the mass matrix), leaving only integrals over Ω_{2D}.

For nonlinear materials μ depends on $\mathbf{B} = \text{curl}\mathbf{A}$, which changes over time and in general varies with z. To account for this, at each integration point IP in Ω_{2D} (2) is used to obtain $\mathbf{B}$ restricted to the cell $C_{IP} := IP \times [-\frac{d}{2}, \frac{d}{2}]$, denoted by $\mathbf{B}_{IP}$, see Fig. 2. In each update, the integration in z is carried out numerically to obtain the averaged parameters in the 2D/1D formulation. To solve the nonlinear system, the fixed point iteration presented in [8] is used.

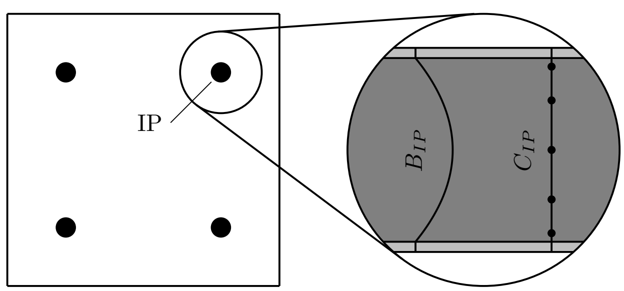

Fig. 2. In each integration point IP the corresponding $\mathbf{B}$ field in z direction is reconstructed and numerical integration is carried out for the terms containing the nonlinear parameter μ.

IV. Numerical Example

The eddy current problem is solved for one sheet of the machine visualized in Fig. 1. The rotor part has an inner radius of 45 mm and an outer radius of 148 mm. It is separated from the stator by a 1.2 mm air gap. The outer radius of the stator is 244 mm. The thickness of the iron sheet is $d = 0.5$ mm. The electric conductivity is given as $\sigma = 2.08 \times 10^6$ S/m and the machine is operated at 1500 rotations per minute.

Both the three dimensional reference solution and the 2D/1D solution use finite element spaces of second order. In the three dimensional case only one half of the iron sheet has to be modeled. However, Tab. I shows that the 2D/1D method still needs only a tenth of the degrees of freedom compared to the reference solution.

TABLE I
Number of degrees of freedom (nDof) for each method

	3D	**3D half domain**	**2D/1D**
nDof	3 445,494	2 122,416	211,361

In Fig. 3 the calculated $\mathbf{B}$ field is compared for both the reference solution and the 2D/1D solution, showing an excellent agreement at a significantly reduced computational cost. The aim of the final paper is to measure the accuracy of this nonlinear model by comparison with measurement data of the simulated machine.

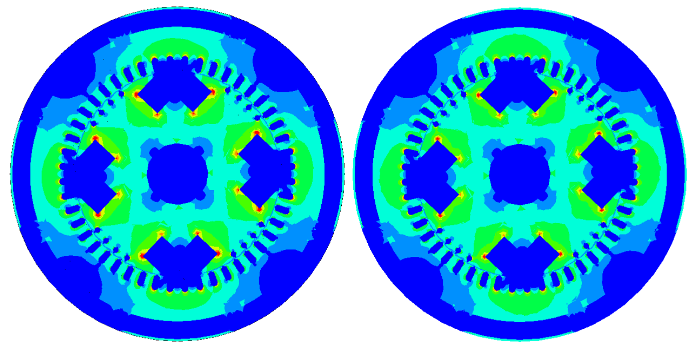

Fig. 3. Comparison of $\mathbf{B}$ for the three dimensional reference solution (left) and the 2D/1D solution (right).

V. Acknowledgment

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