# MSFEM to Simulate Eddy Currents in Laminated Iron

Karl Hollaus[1] and Joachim Schöberl[1]

Vienna University of Technology, Institute for Analysis and Scientific Computing, Wiedner Hauptstrasse 8-10 A-1040 Vienna, Austria karl.hollaus@tuwien.ac.at, joachim.schoeberl@tuwien.ac.at

**Summary.** The simulation of eddy currents in laminated nonlinear iron cores by the finite element method is of great interest in the design of electrical devices. The dimensions of the iron core and the thickness of the laminates are very different. Thus, finite element models considering each laminate require many finite elements leading to extremely large systems of equations. A multi-scale finite element method has been developed to cope with this problem. Numerical simulations in 2D demostrate an excellent accuracy and very low computational costs.

## 1 Introduction

The simulation of the eddy current losses in laminated iron cores is still a challenging task [1]. Brute force methods apply for instance an anisotropic conductivity [2]. These solutions are frequently corrected in a second step [3]. The method proposed in [4] imposes the magnetic properties in a weak sense using skin effect sub-basis functions. The multi-scale finite element method (MSFEM) in [5,6] is extended by higher order micro-shape functions in this work.

## 2 Standard Eddy Current Problem

### 2.1 Boundary Value Problem

In the standard eddy current problem each laminate is resolved by finite elements. The eddy current problem to be solved is sketched in Fig. 1. It consists of a conducting domain (iron) $\Omega_c$ and air $\Omega_0$, i.e., $\Omega = \Omega_c \cup \Omega_0$ with the boundary $\Gamma = \Gamma_D \cup \Gamma_N$. The material parameters are the magnetic permeability $\mu(\boldsymbol{A})$ and the electric conductivity $\sigma$, respectively. The eddy current problem with the magnetic vector potential $\boldsymbol{A}$ in the time domain reads as

$$\mathbf{curl}\,\frac{1}{\mu(\boldsymbol{A})}\,\mathbf{curl}\,\boldsymbol{A} + \sigma\frac{\partial}{\partial t}\boldsymbol{A} = \boldsymbol{J}_0 \text{ in } \Omega \subset \mathbb{R}^3, \quad (1)$$

$$\boldsymbol{A} \times \boldsymbol{n} = \boldsymbol{\alpha} \text{ on } \Gamma_D, \quad (2)$$

$$\frac{1}{\mu(\boldsymbol{A})}\,\mathbf{curl}\,\boldsymbol{A} \times \boldsymbol{n} = \boldsymbol{K} \text{ on } \Gamma_N, \quad (3)$$

where $\boldsymbol{J}_0$ in (1) stands for an impressed current density, $\boldsymbol{\alpha}$ in (2) represents a magnetic flux and $\boldsymbol{K}$ in (3) descripes a surface current density.

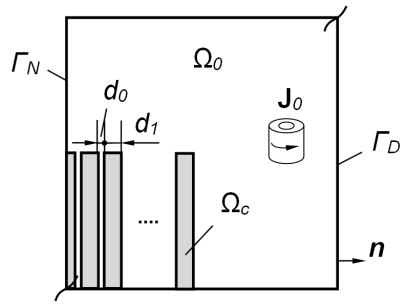


**Fig. 1.** Standard boundary value problem.

### 2.2 Weak Form

Equations (1) to (3) lead to the following weak form for the finite element method (FEM). Find $\boldsymbol{A}_h \in \mathcal{V}_\alpha := \{\boldsymbol{A}_h \in \mathcal{V}_h : \boldsymbol{A}_h \times \boldsymbol{n} = \boldsymbol{\alpha}_h \text{ on } \Gamma\}$, such that

$$\int_\Omega \frac{1}{\mu(\boldsymbol{A}_h)}\,\mathbf{curl}\,\boldsymbol{A}_h \cdot \mathbf{curl}\,\boldsymbol{v}_h\, d\Omega + \frac{\partial}{\partial t}\int_\Omega \sigma \boldsymbol{A}_h \cdot \mathbf{v}_h\, d\Omega = \int_\Omega \boldsymbol{J}_0 \cdot \boldsymbol{v}_h + \int_{\Gamma_N} \boldsymbol{K} \cdot \boldsymbol{v}_h \quad (4)$$

for all $\boldsymbol{v}_h \in \mathcal{V}_0 := \{\boldsymbol{v}_h \in \mathcal{V}_h : \boldsymbol{v}_h \times \boldsymbol{n} = \boldsymbol{0} \text{ on } \Gamma_D\}$, where $\mathcal{V}_h$ is a finite element subspace of $H(\mathbf{curl}, \Omega)$.

## 3 Multi-Scale Finite Element Method MSFEM

### 3.1 Multi-Scale Method

Standard FEM provides accurate approximations as long as the unknown solution is smooth. To avoid large equation systems for equations with rough coefficients the standard polynomial basis is augmented by special functions including a priori information into the ansatz space

$$u^h(x) = \sum_i^n \sum_j^m u_{ij}\varphi_i(x)\psi_j(x) = \sum_i^n \sum_j^m u_{ij}\phi_{ij}(x), \quad (5)$$

where $n$ is the number of standard polynomials $\varphi_i$, $m$ the number of special functions $\psi_j$ and $u_{ij}$ are the coefficients of the approximated solution $u^h$. The special functions representing a local basis approximate well the solution locally [7]. Based on (5) a MSFEM for the eddy current problem with laminated iron has been constructed. The MSFEM and the used local basis are presented below.

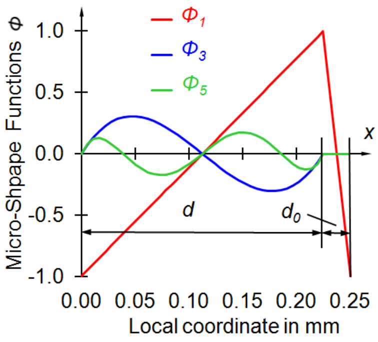


**Fig. 2.** Micro-shape functions.

### 3.2 Micro-Shape Function Basis for MSFEM

The micro-shape functions considering the periodic structure of a laminated stack are shown in Fig. 2. The tooth-shaped function $\phi_1$ is continuous and piecewise linear. The thicknesses of iron layers and air gaps are $d$ and $d_0$, respectively. The feasibility considering only $\phi_1$ with respect to the penedration depth have been shown in [6, 8]. Gauss-Lobatto shape functions were selected for the higher order micro-shape functions. Fig. 2 shows them transformed into the intervall $[0,d]$. The higher order micro-shape functions $\phi_3$ and $\phi_5$ represent bubble functions and are equal to zero in the air gap. Thus, $\{\phi_1, \phi_3, \phi_5\}$ span a subspace of the periodic and continuous functions $H^1_{per}(\Omega)$.

### 3.3 Multi-Scale Approach

Standard polynomial basis functions [9] are augmented by the micro-shape functions leading to the multi-scale approach

$$\tilde{\boldsymbol{A}} = \boldsymbol{A}_0 + \sum_{i\in\{1,3,5\}} \left(\phi_i \left(0,\ A_{i,2},\ A_{i,3}\right)^T + \nabla(\phi_i w_i)\right) \quad (6)$$

where the lamination is perpendicular to the x-direction. The mean value $\mathbf{A_0}$ considers the smooth variation of the marcro-structure, the scalar functions $A_{i,2h}$, $A_{i,3h}$, $w_i$ with $\phi_i$, respectively, take into acount of the periodic micro-structure of the laminated iron. The main magnetic flux density is an even function across a laminate. Consequently, the variation of $\boldsymbol{A}$ is an odd function. Thus, it suffices to consider only odd micro-shape functions in approach (4). An extension of (6) to higher order again is straightforward.

### 3.4 Weak Form of MSFEM

Replacing $\boldsymbol{A}$ in (1) to (3) by (6) leads to the weak form: Find $(\boldsymbol{A}_{0h}, A_{i,2h}, A_{i,3h}, w_{i,h}) \in V_\alpha := \{(\boldsymbol{A}_{0h}, A_{i,2h}, A_{i,3h}, w_{i,h}) : \boldsymbol{A}_{0h} \in \mathcal{U}_h,\ (A_{i,2h}, A_{i,3h}) \in \mathcal{V}_h,\ (w_{i,h}) \in \mathcal{W}_h$ and $\boldsymbol{A}_{0h} \times \boldsymbol{n} = \alpha_h$ on $\Gamma\}$, such that

$$\int_\Omega \frac{1}{\mu(\tilde{\mathbf{A}}_h)} \mathbf{curl}\,\tilde{\mathbf{A}}_h \cdot \mathbf{curl}\,\tilde{\mathbf{v}}_h \, d\Omega + \sigma \frac{\partial}{\partial t} \int_\Omega \sigma \tilde{\mathbf{A}}_h \cdot \tilde{\mathbf{v}}_h \, d\Omega = \int_\Omega \boldsymbol{J}_0 \cdot \tilde{\mathbf{v}}_h + \int_{\Gamma_N} \boldsymbol{K} \cdot \tilde{\mathbf{v}}_h \quad (7)$$

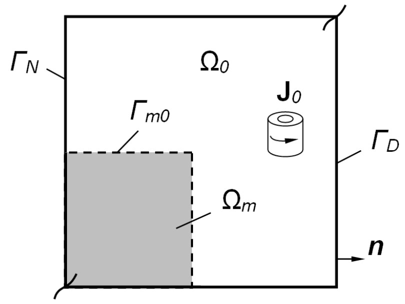


**Fig. 3.** Multi-scale boundary value problem.

for all $(\boldsymbol{v}_{0h}, v_{i,2h}, v_{i,3h}, q_{i,h}) \in V_0 := \{(\boldsymbol{v}_{0h}, v_{i,2h}, v_{2,3h}, q_{i,h}) : \boldsymbol{v}_{0h} \in \mathcal{U}_h,\ (v_{i,2h}, v_{i,3h}) \in \mathcal{V}_h,\ (q_{i,h}) \in \mathcal{W}_h$ and $\boldsymbol{v}_{0h} \times \boldsymbol{n} = \boldsymbol{0}$ on $\Gamma_D\}$, where $\mathcal{U}_h$ is a finite element subspace of $H(\mathbf{curl}, \Omega)$, $\mathcal{V}_h$ of $L_2(\Omega_m)$ and $\mathcal{W}_h$ of $H^1(\Omega_m)$, respectively, and $\{\phi_1, \phi_3, \phi_5\}$ is a subspace of $H_{per}(\Omega_m)$. The subdomain $\Omega_m$ comprises the iron laminates and the air gaps in between as indicated in Fig. 3. Natural boundary conditions hold on the interface $\Gamma_{m0}$. The main magnetic flux was studied in 2D in [8]. Here, the magnetic stray flux is also taken into account in 3D. The orientation of the lamination can vary arbitrarily in space [5].

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