MOR for the MSFEM of the Eddy Current Problem in Linear Laminated Media ★

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1. INTRODUCTION

The simulation of the eddy currents in electrical devices with the finite element method (FEM) is satisfactory. However, the large systems to be solved result in high computational costs, i.e. memory requirement and computation time. Although the multiscale finite element method (MSFEM) can be exploited to simulate eddy currents in laminated iron more efficiently the complexity of the problems are still too large to solve them conveniently. The computational costs are a multiple of the costs of anisotropic models in brute force methods according to the components used in the multiscale formulation, compare with Hollaus and Schöberl (2017).

Model order reduction (MOR) has proven to be a powerful methodology to reduce the costs and is well established for linear problems. MOR with proper orthogonal decomposition (POD) has been applied to solve large scale linear problems in computational electromagnetics very successful. Strategies to select an optimal number of snapshots except those with the largest singular values can be found in Sato and Igarashi (2013) and Klis et al. (2016). Those MOR methods are interesting which exploit properties of specific problems. Splitting of the domain into a region where the solution changes strongly due to a parameter variation and the rest, MOR is applied to the rest with almost constant solution in Sato et al. (2016). For example, the speedup factor is about 1.6 for quasitatic problems in 2D by MOR with POD applied only to the linear domain in Schmidthäusler et al. (2014). MOR is frequently used to facilitate the simulation of electrical machines, see for example [Farzamfar et al. (2017)].

In the present work, the idea is to exploit the specific structure of systems coming from the MSFEM for the eddy current problem (ECP) in laminated media for MOR. For example, the entire problem region can be subdivided into air and the laminated media on the one hand and, on the other, the total solution is composed of a large scale and fine scale part. This work focuses on the second aspect which will be called structural model order reduction (SMOR), see also Klis et al. (2016).

The aim is to study the feasibility to exploit the structure of specific systems arising out of MSFEM of ECPs with laminated media for MOR. Much more accurate results are expected by MSFEM with MOR than by FEM with MOR with the same effort.

First, the basic ECP studied in the present work uses a single component current vector potential (SCCVP) T and is discussed in Sec. 2. Then, MSFEM for T is introduced. Next, MOR and structural model order reduction (SMOR) are explained briefly in Sec. 3. A comparison of numerical results obtained by MOR and SMOR are presented in Sec. 4.

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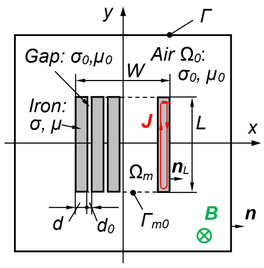

Fig. 1. Eddy current problem in 2D.

2. HIGHER ORDER MSFEM WITH THE SINGLE COMPONENT CURRENT VECTOR POTENTIAL T

2.1 *Boundary value problem with T*

A current vector potential $\boldsymbol{T}$ can be introduced by $\boldsymbol{J} = \operatorname{curl} \boldsymbol{T}$ fulfilling $\operatorname{div} \boldsymbol{J} = 0$ exactly. This work deals with the single component current vector potential T, e.g., pointing in z-direction $\boldsymbol{T} = T\boldsymbol{e}_z$ in the frequency domain. A simple boundary value problem (BVP) of the ECP in the frequency domain reads, see Fig. 1:

$$\operatorname{curl} \frac{1}{\sigma} \operatorname{curl} T + j\mu\omega T = 0 \text{ in } \Omega \subset \mathbb{R}^2 \tag{1}$$

$$T = T_0 \text{ on } \Gamma \tag{2}$$

2.2 *Weak form with T*

The weak form for the FEM in the frequency domain reads:
Find $T_h \in V_{h,T_0} := \{T_h \in \mathcal{U}_h : T_h = T_0 \text{ on } \Gamma\}$, such that

$$\int_\Omega \frac{1}{\sigma} \operatorname{curl} T_h \cdot \operatorname{curl} t_h \, d\Omega + j\omega \int_\Omega \mu T_h t_h \, d\Omega = 0 \tag{3}$$

for all $t_h \in V_{h,0}$, where $\mathcal{U}_h \subset H^1(\Omega)$.

2.3 *Higher order multiscale finite element method with T*

The multiscale approach up to the order 4 for the single component current vector potential

$$\tilde{T}(x,y) = T_0(x,y) + \phi_2(x)T_2(x,y) + \phi_4(x)T_4(x,y) \qquad (4)$$

is considered with even micro-shape functions ϕ_2 and ϕ_4 shown in Fig. 2. Simply speaking T corresponds to the magnetic field strength $\boldsymbol{H}$ which is an even function in the lamination, therefore the micro-shape functions ϕ_2 and ϕ_4 are used in (4).

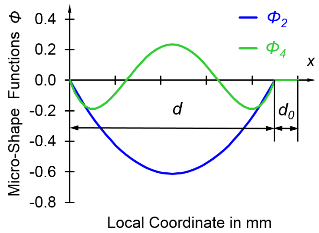

Fig. 2. Even micro-shape functions ϕ_2 and ϕ_4.

2.4 *Weak form of MSFEM with T*

The weak form reads as:
Find $(T_{0h}, T_{2h}, T_{4h}) \in V_{h,T_0} := \{(T_{0h}, T_{2h}, T_{4h}) : T_{0h} \in \mathcal{U}_h, T_{2h}$ and $T_{4h} \in \mathcal{V}_h, T_{0h} = T_0$ on Γ and $T_{2h} = 0$ and $T_{4h} = 0$ on $\Gamma_{m0,1} \subset \Gamma_{m0}\}$, such that

$$\int_\Omega \frac{1}{\sigma} \operatorname{curl} \tilde{T}_h \cdot \operatorname{curl} \tilde{t}_h \, d\Omega + j\omega \int_\Omega \mu \tilde{T}_h \tilde{t}_h \, d\Omega = 0 \qquad (5)$$

for all $(t_{0h}, t_{2h}, t_{4h}) \in V_{h,0}$, where $\mathcal{U}_h$ is a subspace of $H^1(\Omega)$, $\mathcal{V}_h$ of $H^1(\Omega_m)$ and ϕ_2 and $\phi_4 \in H^1_{per}(\Omega_m)$.

3. MOR AND SMOR

Assume that the MSFEM (5) results in the linear equation system

$$Ax = f. \qquad (6)$$

Furthermore, m snapshots x_i, i.e. solutions of $A_i x_i = f$ by modifying a parameter are calculated and inserted as column vectors in the snapshot matrix S with dimension $n \times m$, where usually $n \gg m$ holds. The present work uses the relative permittivity μ_r as parameter. Next, for the POD based MOR a singular value decomposition (SVD)

$$S = U\Sigma V^*, \qquad (7)$$

the star marks conjugate transpose of V, is carried out. Matrices U $(n \times n)$ and V $(m \times m)$ are Hermitian matrices. The singular values σ_i are arranged in the diagonal of Σ with $\sigma_i \geq \sigma_{i+1}$. Now, an appropriate reduced basis

$$W = [u_1\sigma_1, u_2\sigma_2, ..., u_r\sigma_r], \qquad (8)$$

matrix W represents the projection matrix, is selected considering the essential singular values σ_i, where $r \geq m$ is valid. With $x = Wy$ the reduced order model

$$W^T A W y = W^T f = K y = g \qquad (9)$$

is obtained. Similarly, SVDs are carried out of all partitions S_i, where $S = (S_0, S_2, S_4)^T$, according to the unknowns T_0, T_2 and T_4 in the approach (4). Therefore, SMOR yields a larger reduced order model than MOR.

4. NUMERICAL RESULTS

The model shown in Fig. 1 consists of 10 laminates, $d = 1.8$mm, and air gaps in between, $d_0 = 0.2$mm. The dimensions of the domains are $|\Omega_m| = 20 \times 20$ mm^2 and $|\Omega| = 40 \times 40$ mm^2. The frequency f was chosen with 50Hz and the conductivity σ with $2 \cdot 10^6$S/m.

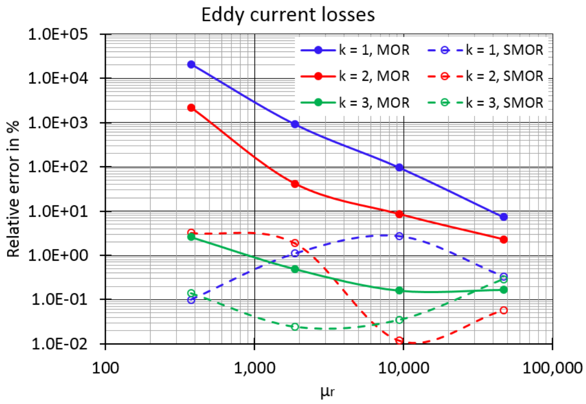

Fig. 3. Comparison of model order reduction (MOR) with structural model order reduction (SMOR).

The relative error presented in Fig. 3 is defined by comparing the eddy current losses P obtained by MOR or SMOR with those of MSFEM:

$$Relative\ error\ in\ \% = \frac{P_{(S)MOR} - P_{MSFEM}}{P_{MSFEM}} \cdot 100 \qquad (10)$$

For the snapshots, μ_r has been selected with 125, 625, 3125, 15625 and 78125. The solutions in Fig. 3 are calculated at μ_r equals 375, 1875, 9375 and 46875, i.e. $m = 5$. The number of basis vectors used in the reduced basis is denoted by k. SMOR provides already for a very small reduced basis reasonable results. The error of MOR decreases for increasing μ_r clearly. MOR and SMOR reduce the MSFEM system by factor of about 100.

5. CONCLUSION

SMOR seams to be working properly already with very few basis vectors, i.e. low dimension of the reduced basis. An extension of SMOR to large and nonlinear problems in 3D will be studied in the future.

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