A Multiscale FEM for the Eddy Current Problem with $T, \Phi - \Phi$ in Laminated Conducting Media

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To avoid the necessity of modeling each laminate in iron cores of electrical devices for the eddy current problem a novel multiscale finite element method based on a current vector potential *T* and a reduced magnetic scalar potential Φ is presented for three-dimensional problems. Material properties are assumed to be linear. Hence, the methods are developed for the frequency domain. External currents are represented by the Biot-Savart-field serving as excitation. Planes of symmetry are exploited. Numerical simulations are presented showing very satisfactory results.

***Index Terms*—Biot-Savart-field, current vector potential, eddy current problem, laminated media, multiscale finite element method, reduced magnetic scalar potential.**

I. Introduction

An accurate prediction of the eddy current distribution in laminated iron cores of electric devices is a challenging task in the design process. Modeling of each laminate requires many finite elements leading to extremely large equation systems. The computational costs to solve these systems are prohibitively high.

The solution obtained by prescribing a current vector potential (CVP) $\boldsymbol{T}$ having a single component normal to the lamination [1] or using anisotropic electric conductivity [2] has to be corrected in a post-processing step to consider the effect of the main magnetic flux on the total eddy current losses. These approaches are questionable in the context of nonlinear material properties. Multiscale finite element methods (MSFEMs) provide the solution in one step taking account of both the main magnetic flux parallel to the lamination and a magnetic stray flux perpendicular to the lamination.

The capacity of the MSFEM is well known [3]. Recently a MSFEM in 3D for eddy currents in laminated iron cores based on the magnetic vector potential $\boldsymbol{A}$ has been presented in [4]. A CVP $\boldsymbol{T}$ [5] with a magnetic scalar potential Φ can also be used to model an eddy current problem (ECP), [6]. The $\boldsymbol{T}, \Phi - \Phi$ formulation is popular to simulate the eddy currents for instance in the core of transformers. To improve the local approximation the magnetic flux density parallel to the lamination is expanded into orthogonal even polynomials, so-called skin effect sub-basis functions, in [7].

The developed approaches for the MSFEMs are presented. Eddy current losses obtained by the new MSFEM have been compared with those obtained by reference solutions of finite element models considering each laminate. A numerical example has been studied to show the the satisfactory performance of the methods.

II. Boundary Value Problem with $\boldsymbol{T}, \Phi - \Phi$

The eddy current problem to be solved is sketched in Fig. 1. It consists of laminates, the conducting domain Ω_c, enclosed by air Ω_0, i.e., $\Omega = \Omega_c \cup \Omega_0$. The CVP $\boldsymbol{T}$ and the reduced magnetic scalar potential Φ are introduced extremely shortened subsequently. Considering Ampere's law $\operatorname{curl} \boldsymbol{H} = \boldsymbol{J} - \boldsymbol{J}_0$ with an impressed current density $\boldsymbol{J}_0$, the magnetic field strength

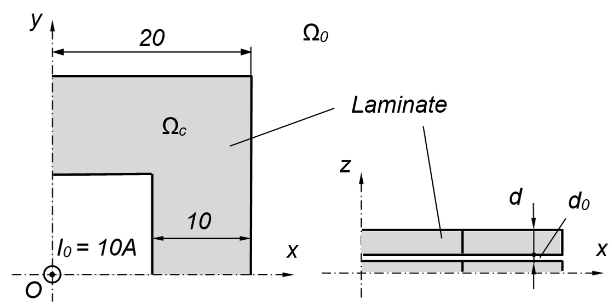

Fig. 1. Eddy current problem, problem not drawn to scale, top view (left) and front view (right), dimensions are in mm, origin O at (0,0,0), planes of symmetry are $x = 0$, $y = 0$ and $z = 0$, quadratic structure in the xy-plane.

$$\boldsymbol{H} = \boldsymbol{T} + \boldsymbol{T}_{BS} - \operatorname{grad} \Phi \quad \text{in } \Omega_c \tag{1}$$

can be represent by a current vector potential $\boldsymbol{T}$ and a reduced magnetic scalar potential Φ, where $\boldsymbol{J}_0$ is replaced by its Biot-Savart-field $\boldsymbol{T}_{BS}$. The potentials $\boldsymbol{T}$ and Φ describe the quasi-static field in Ω_c. The static magnetic field can be written as

$$\boldsymbol{H} = \boldsymbol{T}_{BS} - \operatorname{grad} \Phi \quad \text{in } \Omega_0. \tag{2}$$

Thus, the following boundary value problem for the $\boldsymbol{T}, \Phi - \Phi$ formulation is obtained [6].

Quasi-static magnetic field in the conducting domain Ω_c:

$$\begin{aligned} \operatorname{curl}(\rho \operatorname{curl} \boldsymbol{T}) + j\omega\mu\boldsymbol{T} &- j\omega\mu \operatorname{grad} \Phi = \\ -\operatorname{curl}(\rho \operatorname{curl} \boldsymbol{T}_{BS}) &- j\omega\mu\boldsymbol{T}_{BS}, && (3) \\ j\omega \operatorname{div}(\mu\boldsymbol{T} - \mu \operatorname{grad} \Phi) &= -j\omega \operatorname{div}(\mu\boldsymbol{T}_{BS}) \quad \text{in } \Omega_c && (4) \end{aligned}$$

$$\begin{aligned} \rho \operatorname{curl} \boldsymbol{T} \times \boldsymbol{n} &= -\rho \operatorname{curl} \boldsymbol{T}_{BS} \times \boldsymbol{n}, && (5) \\ \mu (\boldsymbol{T} - \operatorname{grad} \Phi) \cdot \boldsymbol{n} &= -\mu\boldsymbol{T}_{BS} \cdot \boldsymbol{n} \quad \text{on } \Gamma_E && (6) \end{aligned}$$

$$\begin{aligned} \boldsymbol{T} \times \boldsymbol{n} &= -\boldsymbol{T}_{BS} \times \boldsymbol{n} \; (= \boldsymbol{0}), && (7) \\ \Phi &= \Phi_0 \; (= 0) \quad \text{on } \Gamma_{H_c} && (8) \end{aligned}$$

Static magnetic field in the non-conducting domain Ω_0:

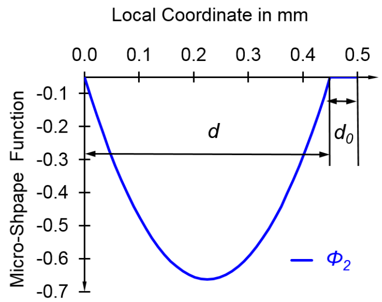

Fig. 2. The micro-shape function ϕ_2.

$$
\begin{aligned}
-j\omega\,\mathrm{div}(\mu\,\mathrm{grad}\,\Phi) &= -j\omega\,\mathrm{div}(\mu \boldsymbol{T}_{BS}) \quad \text{in } \Omega_0 \qquad (9)\\
-\mu \boldsymbol{n}\cdot \mathrm{grad}\,\Phi &= -\mu \boldsymbol{T}_{BS}\cdot \boldsymbol{n} \quad \text{on } \Gamma_B \qquad (10)\\
\Phi &= \Phi_0\ (=0) \quad \text{on } \Gamma_{H_0} \qquad (11)
\end{aligned}
$$

Interface conditions on Γ_{0c} vanish and, therefore, are omitted. Indizes E, H, and B mean, that the tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ and the normal component of $\boldsymbol{B}$ equal to zero. The material parameters are the magnetic permeability μ and the electric resistivity ρ, j means the imaginary unit and ω the angular frequency.

III. Multiscale Approaches

The multiscale approaches (12) and (13) are based on the fact that the problem can be observed as a macro-structure of the iron bulk with large dimensions, on the one hand, and on the other, the micro-structure with the very small thickness of the laminates d and the width of the air gaps d_0 in between (Fig. 1). The mean values $\mathbf{T}_0$ and Φ_0 consider the large scale variations of the solution and $\mathbf{T}_2$ and Φ_2 with the periodic micro-shape functions ϕ_2, see Fig. 2, the highly oscillating variation of the solution due to the micro-structure:

$$
\begin{aligned}
\tilde{\boldsymbol{T}} &= \boldsymbol{T}_0 + \phi_2 \boldsymbol{T}_2, \qquad (12)\\
\tilde{\Phi} &= \Phi_0 + \phi_2 \Phi_2, \qquad (13)
\end{aligned}
$$

where $\boldsymbol{T}_0, \boldsymbol{T}_2 \in H(\mathrm{curl}, \Omega_c)$ and $\Phi_0 \in H^1(\Omega)$ and $\Phi_2 \in H^1(\Omega_c)$. The tilde marks the multiscale approach. Depending on whether (12) is only exploited or both (12) and (13) are employed, two different methods are obtained. The weak formulations for the associated MSFEMs will be presented in the full paper.

IV. Numerical Simulations

The problem consists of three laminates, compare with Fig. 1. The thickness of the laminates d has been selected as 0.45mm and unfavorable air gaps with a width of $d_0 = 0.05$mm have been assumed. A conductivity of $\sigma = 2 \cdot 10^6$S/m and a relative permeability of $\mu_r = 1,000$ were chosen. For the sake of simplicity, the problem is excited by a filamentary current along the z-axis. The reference solution obtained by finite element models considering each laminate individually is denoted by RS. The losses obtained for different frequencies are summarized in Tab. I for one eighth of the problem. The developed MSFEMs have been studied, where method A1 uses only (12) and A2 both (12) and (13). The agreement of the losses is very satisfactory. This remarkable agreement is also confirmed by the field distribution in Figs. 3 and 4.

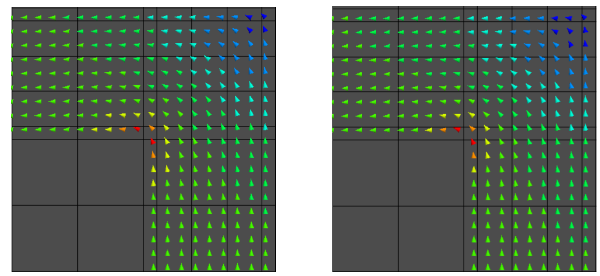

Fig. 3. Real part of the magnetic flux density at y = const. and f = 50Hz, RS (left), MSFEM (right).

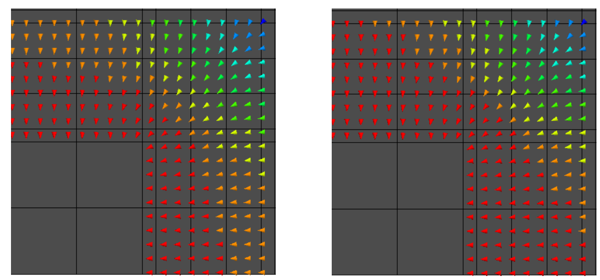

Fig. 4. Imaginary part of the current density at y = const. and f = 50Hz, RS (left), MSFEM (right).

TABLE I
Eddy Current Losses in μW.

f in Hz	RS	$A1$	$A2$
50	4.39	4.19	4.19
500	428	409	409

Its worth to be mentioned that no significant difference between the methods A1 and A2 have been observed.

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