MSFEM and MOR to Minimize the Computational Costs of Nonlinear eddy Current Problems in Laminated Iron Cores

K. Hollaus, J. Schöberl and M. Schöbinger

Technische Universität Wien, Institute for Analysis and Scientific Computing, A-1040 Vienna, karl.hollaus@tuwien.ac.at

The multiscale finite element method (MSFEM) reduces the computational costs for the simulation of eddy currents (ECs) in laminated iron cores compared to the standard finite element method (SFEM) essentially. Nevertheless, the complexity of the resulting problem is still too large to solve it conveniently. The idea is to additionally exploit model order reduction (MOR). Snapshots for a reduced basis are calculated by MSFEM cheaply. Numerical simulations of a small transformer show an exceptional performance.

***Index Terms*—Laminated iron core, multiscale finite element method MSFEM, model order reduction MOR, nonlinear eddy current problem, time stepping method.**

I. INTRODUCTION

AN ACCURATE and efficient simulation of the ECs in laminated iron cores by the FEM is of great interest in the design of electrical devices. However, the dimensions of such cores are extremely different. The overall dimensions, i.e. the large scale, are in the range of meters whereas the small scale, determined by the thickness of the laminates and the width of gaps between them, is in the sub-millimeter range. Moreover, the magnetic properties of iron are highly nonlinear. Therefore, modeling of each laminate of iron cores by finite elements would lead to extremely large nonlinear systems of equations impossible to solve with present computer resources reasonably. The MSFEM makes use of the quasi-periodic micro-structure of laminated cores and reduces the computational costs essentially without loosing accuracy [1], [2] compared to the SFEM.

Although, the MSFEM has brought a great progress in solving the eddy current problem (ECP) in laminated cores, the complexity is still too large to become a routine task for engineers. The MOR has proven to be a powerful tool to reduce the costs drastically and is well established for linear problems in computational electromagnetics. Proper orthogonal decomposition based MOR uses the method of snapshots to select an optimal basis for the reduced model. This has been applied to solve large scale linear problems very successfully, e.g. [3]. However, nonlinear problems are still extremely challenging [4], [5]. The by far more demanding nonlinear ECPs are currently under intensive investigations, see [6], [7], [8].

The aim is to simulate the nonlinear ECP as accurate as possible at minimal computational costs. The idea of this work is to exploit the MSFEM for laminated cores to compute the snapshots for the reduced basis of the large nonlinear problem with reasonable effort. A small single phase transformer is studied. The numerical results are excellent.

II. EDDY CURRENT PROBLEM ECP

The boundary value problem to be solved for the numerical example in Sec. V is the nonlinear ECP

$$\begin{aligned} \operatorname{curl}\mu^{-1}(\mathbf{B})\operatorname{curl}\mathbf{A} + \sigma\frac{\partial}{\partial t}\mathbf{A} &= \boldsymbol{J}_0 \quad \text{in } \Omega = \Omega_c \cup \Omega_0, \\ \boldsymbol{A}\times\boldsymbol{n} &= \mathbf{0} \quad \text{on } \Gamma_D, \\ \mu^{-1}(\mathbf{B})\operatorname{curl}\boldsymbol{A}\times\boldsymbol{n} &= \mathbf{0} \quad \text{on } \Gamma_N, \end{aligned} \tag{1}$$

where $\boldsymbol{A}$ is the magnetic vector potential, $\boldsymbol{B}$ the magnetic flux density, Ω_c represents the conducting domain (iron) and Ω_0 the non-conducting domain (air). The current density $\boldsymbol{J}_0$ is prescribed in the coils. The boundary conditions in (1) represent also planes of symmetry.

III. THE MULTISCALE FINITE ELEMENT METHOD MSFEM

Studying the EC distribution in a laminated iron core the multiscale approach

$$\tilde{\boldsymbol{A}} = \boldsymbol{A}_0 + \phi_1\boldsymbol{A}_1 + \operatorname{grad}(\phi_1 w_1) \tag{2}$$

can be constructed. Then, simply speaking, approach (2) replaces $\boldsymbol{A}$ in (1) to obtain the MSFEM. A detailed explanation of the corresponding weak form, of the micro-shape function ϕ_1 and the meaning of the components $\boldsymbol{A}_0$, $\boldsymbol{A}_1$ and w_1 can be found in [9].

IV. MODEL ORDER REDUCTION MOR

The MSFEM results in the nonlinear equation system

$$A(\mu)x = f. \tag{3}$$

The dimension of A is $n \times n$. The permeability μ in (3) indicates the non-linearity. A snapshot matrix

$$S = (x_1, x_2, ..., x_m) \tag{4}$$

is determined by snapshots x_i as column vectors, which are solutions of $A_i(\mu)x_i = f_i$ varying the input currents in time feeding the coils, at m time instants. The dimension of S equals $n \times m$, where $n \gg m$ holds.

V. Numerical Results

Only one eighth of a single phase transformer has been simulated due to three planes of symmetry [9]. Two excitation coils are fed by input currents I to study the performance of the technique with respect to different saturations. The implicit Euler method has been used for the time stepping method and a fixed point method to deal with the non-linearity. Investigations in [9] have shown an excellent performance of the MSFEM based on $\boldsymbol{A}$ compared to the standard finite element method (SFEM).

Snapshots are determined with rather large time steps within the first half period in time with MSFEM, see Fig. 1. The influence of the non-linearity is clearly visible in Fig. 1 comparing the losses for different input currents I. The relative error with 4 snapshots is in general small, but large for small losses and sometimes almost at 30%, see Fig. 2. Using 8 snapshots provides an exceptionally small overall relative error. The relative error is almost always well below 1%.

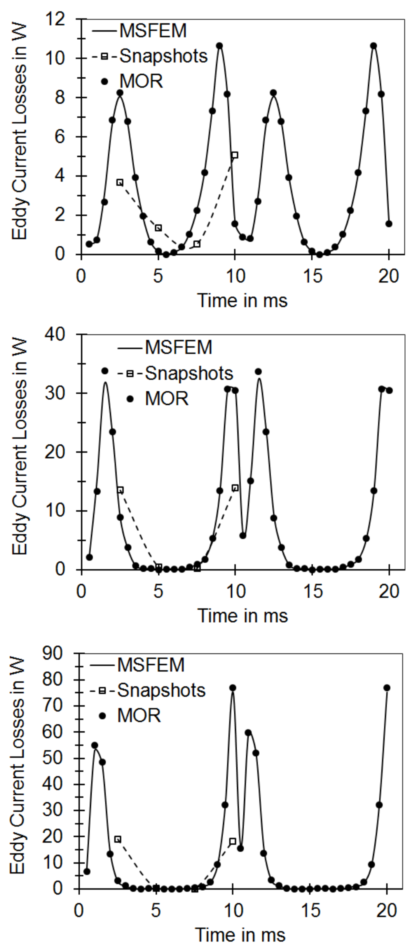

Fig. 1. Eddy current losses for different input currents I=1A (top), I=2A (middle) and I=3A (bottom) at frequency of f=50Hz.

VI. Computational costs

The required number of degrees of freedom (DOFs) are summarized in Tab. I. It shows clearly the reduction of the size of the system of equations to be solved. The total number of nonlinear iterations are 158 for MSFEM only and 175 for

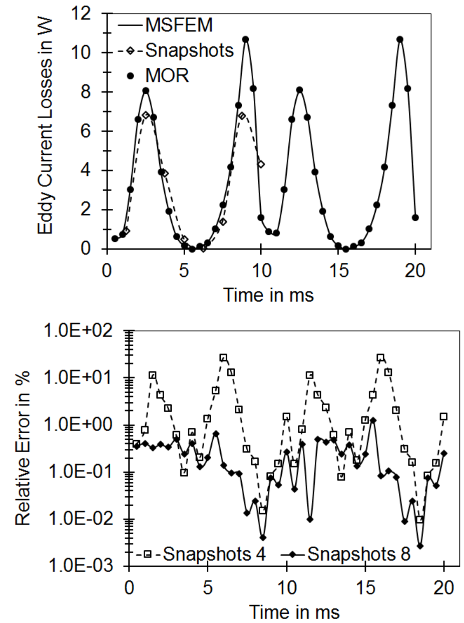

Fig. 2. Eddy current losses with $I = 1A$ (above) and with 8 snapshots. Relative errors (below) for 4 and 8 snapshots.

TABLE I
No. of unknowns DOFs.

Method(s)	SFEM	MSFEM	MSFEM and MOR
DOFs	1 116,860	103,879	4 (8)

MSFEM with MOR with 8 snapshots for 40 time steps and for I=1A for the first period.

Acknowledgment

This work was supported by the Austrian Science Fund (FWF) under Project P 31926.

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