Effective BH-Curves for Laminated Magnetic Cores

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The paper presents a methodology for replacing a laminated magnetic core with an equivalent nonlinear medium in such a way that the reaction field and losses are represented as accurately as possible. This leads to a semi-analytical model with an effective complex-valued BH-curve. The spatial period of the structure does not have to be small relative to the penetration depth.

***Index Terms*—Homogenization, laminated cores, BH-curves, multiscale, eddy currents, losses.**

INsulation-coated laminations are widely used in magnetic cores to reduce eddy current losses. For purposes of simulation and analysis, it is highly desirable to replace the fine structure of the laminations with an approximately equivalent homogeneous material. “Equivalence” implies that reaction field and losses are accurately represented on the coarse (homogenized) scale. This homogenization problem has been extensively studied [2], [3], [5]–[8]. Our approach complements these studies and involves *non-asymptotic* homogenization [11], [12]. We develop a semi-analytical physical model, which can then be solved using any suitable numerical techniques. To fix ideas, we previously focused on the linear case [9]; here we develop a detailed nonlinear model.

One lamination is represented in cylindrical coordinates as a rectangle whose width does *not* have to be small relative to the penetration depth. Details of the setup can be found in [9]. One peculiar feature of the problem is that *eddy currents and related losses cannot be accounted for on the coarse scale* [8], [9]. Counter-intuitively, we define a *magnetostatic* rather than an eddy current problem on the coarse scale.

The magnetic field $\mathbf{h}$ in the original fine structure satisfies the standard eddy current equations, with the displacement current neglected. Under the harmonic balance approximation [1], these equations are $\nabla \times \mathbf{e} = -i\omega\mathbf{b}$, $\nabla \times \mathbf{h} = \mathbf{j}$ with intrinsically isotropic material relations $\mathbf{b} = \mu(|\mathbf{h}|)\mathbf{h}; \mathbf{j} = \sigma\mathbf{e}$. We used the vector potential $\mathbf{t} = (t_z, t_r)$ of eddy currents and the magnetic scalar potential u. This formulation, in both strong and weak form, is very well known [5], [6], [10]. We discretized the fine-scale problem using high-order edge elements for $\mathbf{t}$ and nodal elements for u. On the coarse scale, the BH curve is complex-valued, similarly to [4], and is optimized to accurately represent the reaction field and, via the volume integral of Im BH^*, the losses.

We consider a small cylindrical core and a nonlinear material. As can be seen in Fig. 1, the homogenized solution correctly reproduces the total losses, but the loss distribution is qualitatively affected by the nonlinearity.

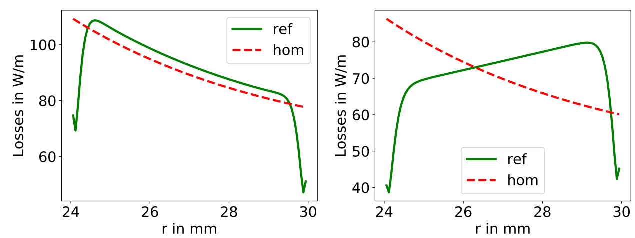

Fig. 1. Losses for the reference solution and the homogenized solution for the linear (left) and highly nonlinear (right) case, integrated over z and θ. Detailed data will be presented in the full paper.

Our non-asymptotic nonlinear homogenization methodology is applicable to laminated magnetic cores with realistic cylindrical geometries. The end result is a coarse-level (homogenized) model with an adjusted complex-valued BH curve representing the reaction field and losses.

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