Multiscale Finite Element Method, Harmonic Balance Method and Model Order Reduction for Nonlinear Eddy Currents in Laminations

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***Abstract*—To obtain a computationally feasible model of an eddy current problem (ECP) in laminated iron cores of electrical devices the multiscale finite element method (MSFEM) based on the magnetic vector potential has been exploited. In the present work the additional use of the harmonic balance method (HBM) and a model order reduction (MOR) method has been studied to allow even more efficient simulations.**

***Index Terms*—Harmonic balance method HBM, laminated iron cores, model order reduction MOR, multiscale finite element method MSFEM, nonlinear ECP.**

I. Numerical Methods

The MSFEM has made an enormous progress in efficient simulations of ECPs in laminated iron cores in recent years [1]. An additional significant step forward has been achieved using MOR along with a time stepping method (TSM) in [2].

Although the TSM is the natural choice for simulations of nonlinear problems, a well known shortcoming of the TSM is sometimes the high number of time steps required for large devices due to slowly decaying transients. That is why in case of time harmonic excitation and steady state the HBM is preferred to avoid expensive simulations.

The time stepping method of a nonlinear ECP and using MOR with snapshots as POD was proposed in [2]. The reduced basis consists of solutions at selected time steps from a time stepping procedure with rather large time steps.

The approach

$$\tilde{\boldsymbol{A}} = \boldsymbol{A}_0 + \phi_1 \boldsymbol{A}_1 + \operatorname{grad}(\phi_1 w_1) \tag{1}$$

for the MSFEM of eddy current current problems has been used. The Fourier expansion of (1) is written as

$$\hat{\boldsymbol{A}} = \boldsymbol{A}_0(\boldsymbol{x}) + \sum_{k=1}^{N} \widetilde{\boldsymbol{A}}^{c}_{2k-1}(\boldsymbol{x}) \cos((2k-1)\omega t) + \widetilde{\boldsymbol{A}}^{s}_{2k-1}(\boldsymbol{x}) \sin((2k-1)\omega t) \tag{2}$$

for the HBM with coefficient functions

$$\widetilde{\boldsymbol{A}}^{\alpha}_{2k-1}(\boldsymbol{x}) = \boldsymbol{A}^{\alpha}_{0,2k-1}(\boldsymbol{x}) + \phi_1 \boldsymbol{A}_{1,2k-1}(\boldsymbol{x}) + \operatorname{grad}\left(\phi_1 w^{\alpha}_{1,2k-1}(\boldsymbol{x})\right), \tag{3}$$

where $\alpha = c, s$ and $k \in \mathbb{N}$, $k \le N$ holds. The time average $\boldsymbol{A}_0(\boldsymbol{x})$ in (2) is not used in this work. The tilde indicates the MSFEM approach and the hat the truncated Fourier expansion of the HBMSFEM approach.

II. Numerical Problem

The numerical problem is shown in Fig. 1. A conductivity of $\sigma = 2 \cdot 10^6 \mathrm{S/m}$ has been assumed. The convex-concave magnetization curve of the steel M400-50A, determined by

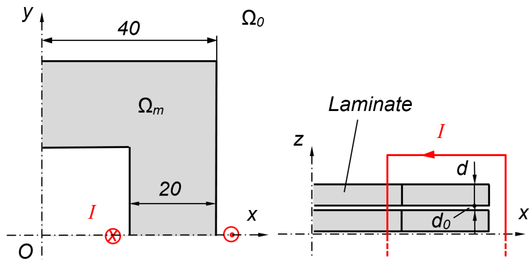

Fig. 1. Numerical example, problem not drawn to scale, origin O at (0,0,0), $x = 0$, $y = 0$ and $z = 0$ represent planes of symmetry, top view (left) and front view (right) of one eighth with four laminates, dimensions are in mm, the structure is quadratic in the xy-plane, rectangular (30×15) current loops are linked symmetrically with the limbs, $d + d_0 = 0.25\mathrm{mm}$ selected with 2.5% fill factor.

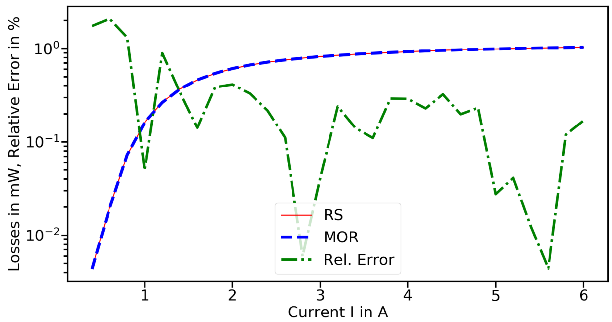

Fig. 2. Losses and relative error obtained by RS and by MOR.

measurement points and linear interpolation, has been used as the nonlinear material model in the simulations.

First, the eddy current losses for currents I of 0.2 to $6.0A$ have been computed in steps of $0.2A$ representing the reference solution (RS). Then, the RSs for I equals 1.0, 3.0 and $5.0A$ have been selected as basis vectors for the MOR method without the subsequent orthogonalization. The losses obtained by the RS and those with MOR are shown in Fig. 2. There is a very good agreement as proven by the relative error.

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References

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