Enhanced Technique for Metascreens using the Generalized Finite Element Method

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***Abstract*—An efficient approach to simulate the multiple scales, occurring in metascreens, is introduced. The standard finite element method (SFEM) leads to a large number of unknowns for metascreens. Therefore, the generalized finite element method (GFEM) is applied with special functions, in this work eigenmodes, with support localized at the apertures. The idea is that this technique is able to cope with varying thicknesses of the metascreen and arbitrary shapes of apertures as well as lossy materials and frequencies in a wide range.**

***Index Terms*—Electromagnetic compatibility, generalized finite element method, metascreens.**

I. Problem Description

In the context of electromagnetic compatibility metallic sheets with small quasi-periodic apertures called metascreens occur [1], see Fig. 1. Here, only the basic cell problem is solved, but the idea is to exploit the cell solution for the quasi-periodic structure of metascreens. The proposed technique leads to an essentially smaller number of unknowns compared to the SFEM.

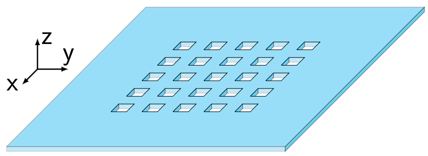

Fig. 1. Metascreen with small periodic apertures.

II. Generalized Finite Element Method

The GFEM uses the SFEM enhanced with special functions [2]. Therefore, the approach for the solution of the GFEM $\mathbf{A}_G$ reads as

$$\mathbf{A}_G = \mathbf{A}_S + \sum_{i=1}^{N} \alpha_i \mathbf{M}_i,$$

where $\mathbf{A}_S$ is an element of the SFEM space and the functions $\mathbf{M}_i$ are the special functions. The idea is to use a coarse discretization for the SFEM resulting in a solution which grasps the global solution but does not resolve the details of the apertures. The local behaviour of the solution is represented by the special functions which are only supported near the apertures. The performance of the GFEM heavily depends on the selection of the special functions. In theory the special functions may be arbitrary functions that are elements of the space in which the solution lives. Because of the quasi-periodicity of the metascreen the special functions are to be chosen periodic and thus are calculated considering only one cell. Here, the special functions used are the eigenmodes of the corresponding eigenvalue problem.

III. Numerical Example

The performance of the GFEM is analysed by means of a representative 2D example with metallic components modelled as perfect electric conductors (PEC). The goal is to solve Maxwell's equations in the frequency domain on one cell via the magnetic vector potential $\mathbf{A}$. The reference solution and the eigenmodes are calculated through SFEM with a high order finite element space. The GFEM uses a coarse SFEM space. The geometry of the 2D example as well as the coarse SFEM solution and the GFEM solution with five eigenmodes can be seen in Fig. 2. The reference solution and the GFEM solution agree very well. The convergence of the relative error in the energy norm with respect to the number of eigenmodes is presented in Tab. I. All numerical simulations have been carried out with NGSolve https://ngsolve.org.

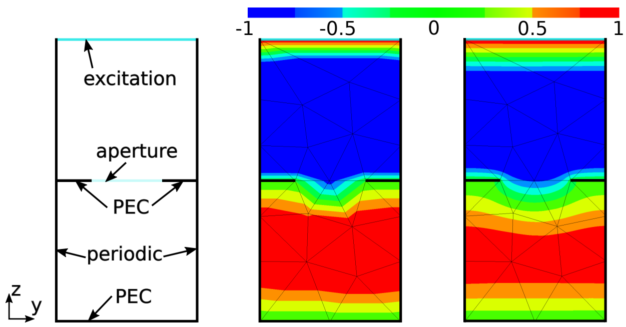

Fig. 2. Geometry (left), coarse SFEM solution $\mathbf{A}$ (middle), GFEM solution $\mathbf{A}_G$ (right).

TABLE I
Convergence GFEM.

Number eigenmodes N	0	1	2	3	4	5
Relative error	81%	69%	8%	7.9%	6%	4.2%

References

[1] C. L. Holloway and E. F. Kuester, "Generalized sheet transition conditions for a metascreen—a fishnet metasurface," *IEEE Transactions on Antennas and Propagation*, vol. 66, no. 5, p. 2414–2427, May 2018.

[2] T. Strouboulis, K. Copps, and I. Babuška, "The generalized finite element method: an example of its implementation and illustration of its performance," *International Journal for Numerical Methods in Engineering*, vol. 47, no. 8, pp. 1401–1417, 2000.