A Mixed Multiscale Finite Element Method for 3D Eddy Currents in Iron Laminates

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***Abstract*—The aim of this work is to study the performance of a mixed multiscale finite element method (MSFEM) for eddy currents in laminated iron in three dimensions. The mixed multiscale approach uses the magnetic vector potential and the current density. The weak form of the mixed MSFEM is presented. Simulations show the capability of the mixed MSFEM to efficiently and accurately approximate eddy currents in laminates.**

***Index Terms*—Eddy currents in 3D, laminated iron cores, mixed multiscale finite element method**

I. Introduction

The magnetic flux density parallel to the lamination is expanded by orthogonal basis functions proposed in the MSFEM [1]. Contrary to [2] the magnetic vector potential $\boldsymbol{A}$ and the current density $\boldsymbol{J}$ are unknown in the present work. The mixed MSFEM considers also the edge effect.

II. MSFEM of the Mixed Formulation with A and J

Assuming the current density in the standard finite element approach

$$\mathbf{J}_h = -j\omega\sigma\mathbf{A}_h \tag{1}$$

also as unknown yields the following mixed weak form.

A. Weak Form of the Mixed FEM with A and J

Find $(\mathbf{A}_h, \mathbf{J}_h) \in V_{h,\alpha} := \{(\mathbf{A}_h, \mathbf{J}_h) : \mathbf{A}_h \in \mathcal{U}_h, \mathbf{J}_{0h} \in \mathcal{M}_h$ and $\boldsymbol{A}_h \times \boldsymbol{n} = \alpha_h$ on $\Gamma\}$, such that

$$\int_\Omega \nu \operatorname{curl} \mathbf{A}_h \cdot \operatorname{curl} \mathbf{v}_h \, d\Omega - \int_\Omega \mathbf{J}_h \cdot \mathbf{v}_h \, d\Omega = 0 \tag{2}$$

$$-\int_\Omega \mathbf{A}_h \cdot \mathbf{g}_h \, d\Omega + \frac{j}{\omega} \int_\Omega \frac{1}{\sigma} \mathbf{J}_h \cdot \mathbf{g}_h \, d\Omega = 0 \tag{3}$$

for all $(\mathbf{v}_h, \mathbf{g}_h) \in V_{h,0}$ with $\mathcal{U}_h \subset H(\operatorname{curl}, \Omega)$ and $\mathcal{M}_h \subset H(\operatorname{div}, \Omega_m)$, where Ω_m is the laminated domain, Ω the entire domain and Γ its boundary.

B. Mixed Multiscale Approach with A and J

The mixed multiscale approach

$$\tilde{\mathbf{A}} = \mathbf{A_0} + \phi_1(0, A_{12}, A_{13})^T + \nabla(\phi_1 w_1) \tag{4}$$

$$\tilde{\mathbf{J}} = \mathbf{J_0} + \operatorname{curl}(\phi_2 \mathbf{T}_2) \tag{5}$$

has been constructed leading to the weak form below.

C. Weak Form of Mixed MSFEM with A and J

Find $(\mathbf{A}_{0h}, A_{12h}, A_{13h}, w_{1h}, \mathbf{J}_{0h}, \mathbf{T}_{2h}) \in V_{h,\alpha} := \{(\mathbf{A}_{0h}, A_{12h}, A_{13h}, w_{1h}, \mathbf{J}_{0h}, \mathbf{T}_{2h}) : \mathbf{A}_{0h} \in \mathcal{U}_h, A_{12h}$ and $A_{13h} \in \mathcal{V}_h$, $w_{1h} \in \mathcal{W}_h$, $\mathbf{T}_{2h} \in \mathcal{U}_h$, $\mathbf{J}_{0h} \in \mathcal{M}_h$ and $\boldsymbol{A}_{0h} \times \boldsymbol{n} = \alpha_h$ on $\Gamma_B\}$, such that

$$\int_\Omega \frac{1}{\mu} \operatorname{curl} \tilde{\mathbf{J}}_h \cdot \operatorname{curl} \tilde{\mathbf{v}}_h \, d\Omega - \int_\Omega \tilde{\mathbf{J}}_h \cdot \tilde{\mathbf{v}}_h \, d\Omega = 0 \tag{6}$$

$$-\int_\Omega \tilde{\mathbf{A}}_h \cdot \tilde{\mathbf{g}}_h \, d\Omega + \frac{j}{\omega} \int_\Omega \frac{1}{\sigma} \tilde{\mathbf{J}}_h \cdot \tilde{\mathbf{g}}_h \, d\Omega = 0 \tag{7}$$

$$p \int_\Omega \operatorname{div} \mathbf{J}_{0h} \operatorname{div} \mathbf{g}_{0h} \, d\Omega = 0 \tag{8}$$

for all $(\mathbf{v}_{0h}, v_{12h}, v_{13h}, q_{1h}, \mathbf{g}_{0h}, \mathbf{t}_{2h}) \in V_{h,0}$ with $\mathcal{U}_h \subset H(\operatorname{curl}, \Omega)$, $\mathcal{V}_h \subset L_2(\Omega_m)$, $\mathcal{W}_h \subset H^1(\Omega_m)$, $\mathcal{M}_h \subset H(\operatorname{div}, \Omega)$, ϕ_1 and $\phi_2 \in H^1_{per}(\Omega_m)$ and for a sufficiently large $p \in \mathbb{R}$.
The micro-scale currents $\operatorname{curl}(\phi_2 \mathbf{T}_2)$ are divergence free. To get a divergence free macro-scale current density $\mathbf{J}_{0h}$ the penalty term is required separately in the weak form. The mixed formulation reproduces the eddy current distribution with the edge effect accurately as can be seen in Fig. 1.

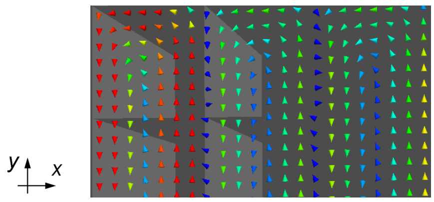

Figure 1: Eddy currents in laminates, detail.

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References

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