Floating Strike Lookback Option

A floating strike lookback option is an option with its strike price set equal to the optimal value that is achieved by the underlying asset over the option’s life. In the case of a call, that optimal value is the lowest value achieved by the underlier during the life of the option, so it pays off the difference between the final value of the underlying asset and that lowest value. In the case of a put, the option pays off the difference between the highest value achieved and the value of the underlying asset at expiration.

The values of floating strike lookback options can be found by the following formulas:

Let $M$ denote the realized minimum, $S$ the initial stock price, $X$ the strike price, $r$ the risk free interest rate, $q$ the dividend rate, $T$ the time to maturity and $\sigma$ the volatility. Then the fair value of a floating strike lookback call is

$$Se^{-q(T-t)}N(d_1) - Me^{-r(T-t)}N(d_2) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-q)} \times \left( \frac{S}{M} \right)^{-2(r-q)/\sigma^2} N\left( -d_1 + \frac{2(r-q)\sqrt{T-t}}{\sigma} \right) - e^{(r-q)(T-t)}N(-d_1)$$

where

$$d_1 = \frac{\log(S/M) + (r - q + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T - t}.$$ 

Let now $M$ denote the realized minimum. With the notation above, the value of a floating strike lookback put is

$$Me^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-q)} \times \left( -\frac{S}{M} \right)^{-2(r-q)/\sigma^2} N\left( d_1 - \frac{2(r-q)\sqrt{T-t}}{\sigma} \right) + e^{(r-q)(T-t)}N(-d_1)$$

where

$$d_1 = \frac{\log(S/M) + (r - q + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T - t}.$$ 

For further details see P.Wilmott: Paul Wilmott On Quantitative Finance, Vol. 1.