Fixed Strike Lookback Option

A fixed strike lookback option has a strike price set in advance. Its exercise depends on the optimal value achieved by the underlying asset during the life of the option. In the case of a call, the optimal value is the highest value the underlier achieves, so the call pays off the difference between that value and the strike price, if positive, and zero otherwise. In the case of a put, the optimal value is the lowest value achieved by the underlying asset and the put pays off the difference between the strike price and that lowest value, if positive, and zero otherwise.

The values of fixed strike lookback options can be found by the following formulas:

Let \( M \) denote the realized maximum of the asset price, \( S \) the initial stock price, \( X \) the strike price, \( r \) the risk free interest rate, \( q \) the dividend rate, \( T \) the time to maturity and \( \sigma \) the volatility.

Then the fair value of a **fixed strike lookback call** is

\[
Se^{-q(T-t)}N(d_1) - Xe^{-r(T-t)}N(d_2) + Se^{-r(T-t)} \frac{\sigma^2}{2(r-q)} \times \left( -\left( \frac{S}{X} \right)^{-2(r-q)/\sigma^2} N\left( d_1 - \frac{2(r-q)\sqrt{T-t}}{\sigma} \right) + e^{(r-q)(T-t)} N\left( d_1 \right) \right)
\]

where

\[
d_1 = \frac{\log(S/X) + (r - q + \sigma^2/2)(T - t)}{\sigma\sqrt{T-t}}
\]

and

\[
d_2 = d_1 - \sigma\sqrt{T-t}
\]

if \( X > M \) and otherwise

\[
(M - X)e^{-r(T-t)} + Se^{-q(T-t)}N(d_1) - Me^{-r(T-t)}N(d_2) + Se^{-r(T-t)} \frac{\sigma^2}{2(r-q)} \times \left( -\left( \frac{S}{M} \right)^{-2(r-q)/\sigma^2} N\left( d_1 - \frac{2(r-q)\sqrt{T-t}}{\sigma} \right) + e^{(r-q)(T-t)} N\left( d_1 \right) \right)
\]

where

\[
d_1 = \frac{\log(S/M) + (r - q + \sigma^2/2)(T - t)}{\sigma\sqrt{T-t}}
\]

and

\[
d_2 = d_1 - \sigma\sqrt{T-t}
\]

Let now \( M \) denote the realized minimum. With the notation above, the value of a **fixed strike lookback put** is

\[
Xe^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1) + Se^{-r(T-t)} \frac{\sigma^2}{2(r-q)} \times \left( -\left( \frac{S}{X} \right)^{-2(r-q)/\sigma^2} N\left( -d_1 - \frac{2(r-q)\sqrt{T-t}}{\sigma} \right) - e^{(r-q)(T-t)} N\left( -d_1 \right) \right)
\]

where

\[
d_1 = \frac{\log(S/X) + (r - q + \sigma^2/2)(T - t)}{\sigma\sqrt{T-t}}
\]

and

\[
d_2 = d_1 - \sigma\sqrt{T-t}
\]
if \( X < M \) and otherwise
\[
(X - M)e^{-r(T-t)} - Se^{-q(T-t)}N(-d_1) + Me^{-r(T-t)}N(-d_2) + Se^{-r(T-t)}\frac{\sigma^2}{2(r-q)}
\times \left( \left( \frac{S}{M}\right)^{-2(r-q)/\sigma^2}N(-d_1 + \frac{2(r-q)\sqrt{T-t}}{\sigma}) - e^{(r-q)(T-t)}N(-d_1) \right)
\]
where
\[
d_1 = \frac{\log(S/M) + (r - q + \sigma^2/2)(T - t)}{\sigma \sqrt{T - t}}
\]
and
\[
d_2 = d_1 - \sigma \sqrt{T - t}.
\]

For further details see P.Wilmott: Paul Wilmott On Quantitative Finance, Vol. 1.