Compound Option

Compound options are options on options. At the first exercise date $T_1$ you must decide whether it is worth exercising the first option (depending on the strike price $X_1$ and the current asset price $S$). If so, you get a further option with strike price $X_2$ and maturity $T_2$.

There are four possible payoffs:

- **Call on call**: $\max\{C(S, T_1) - X_1, 0\}$
- **Call on put**: $\max\{P(S, T_1) - X_1, 0\}$
- **Put on call**: $\max\{X_1 - C(S, T_1), 0\}$
- **Put on put**: $\max\{X_1 - P(S, T_1), 0\}$

The prices of compound options can be found by the following formulas:

The current value of a European call option on a call option is

$$Se^{-qT_2}M(a_1, b_1, \sqrt{T_1/T_2}) - X_2e^{-rT_2}M(a_2, b_2, \sqrt{T_1/T_2}) - e^{-rT_1}X_1N(a_2)$$

where

$$a_1 = \frac{\ln(S/S^*) + (r - q + \sigma^2/2)T_1}{\sigma\sqrt{T_1}}, \quad a_2 = a_1 - \sigma\sqrt{T_1},$$

and

$$b_1 = \frac{\ln(S/X_2) + (r - q + \sigma^2/2)T_2}{\sigma\sqrt{T_2}}, \quad b_2 = b_1 - \sigma\sqrt{T_2}.$$

$M$ denotes the cumulative bivariate normal distribution function and $S^*$ the stock price at $T_1$ for which the option price equals $X_1$. You will exercise the first option only if at $T_1$ the actual stock price is above $S^*$. Furthermore, $r$ is the interest rate, $q$ the dividend rate and $\sigma$ the volatility.

Similarly the value of a European put on a call is

$$X_2e^{-rT_2}M(-a_2, b_2, -\sqrt{T_1/T_2}) - Se^{-qT_2}M(-a_1, b_1, -\sqrt{T_1/T_2}) + e^{-rT_1}X_1N(-a_2).$$

The value of a European call on a put is

$$X_2e^{-rT_2}M(-a_2, -b_2, \sqrt{T_1/T_2}) - Se^{-qT_2}M(-a_1, -b_1, \sqrt{T_1/T_2}) - e^{-rT_1}X_1N(-a_2).$$

The value of a European put on a put is

$$Se^{-qT_2}M(a_1, -b_1, -\sqrt{T_1/T_2}) - X_2e^{-rT_2}M(a_2, -b_2, -\sqrt{T_1/T_2}) + e^{-rT_1}X_1N(a_2).$$

For more details see J.C. Hull: Options, Futures, And Other Derivatives.