ERRATUM ON “ENTROPY-ENERGY INEQUALITIES AND IMPROVED CONVERGENCE RATES FOR NONLINEAR PARABOLIC EQUATIONS”

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There is a gap in the proof of Proposition 1-ii) and Theorem 4-ii) claiming that
\[ \lim_{t \to \infty} \Sigma_k [u(\cdot, t)] = 0. \]

In the following, we explain how this gap can be closed.

Indeed, by a Sobolev-Poincaré inequality and the dissipation of entropy estimate of Lemma 2 or Theorem 3, for some constant \( c > 0 \),
\[
\int_0^\infty \left\| u^{(k+m)/2}(\cdot, s) - \int_{S^1} u^{(m+k)/2}(x, s) \, dx \right\|_{L^\infty(S^1)}^2 \, ds \leq c \int_0^\infty \int_{S^1} \left| (u^{(k+m)/2})_x(x, s) \right|^2 \, dx \, ds < \infty.
\]

Thus, there exists an increasing diverging sequence \( (t_n)_{n \in \mathbb{N}} \to +\infty \) such that
\[
\left\| u^{(k+m)/2}(\cdot, t_n) - \int_{S^1} u^{(m+k)/2}(x, t_n) \, dx \right\|_{L^\infty(S^1)} \to 0 \quad (1)
\]
as \( n \to \infty \). On the other hand, for the same subsequence, we can assume without loss of generality that \( (u^{(k+m)/2})_x(\cdot, t_n) \to 0 \) in \( L^2(S^1) \). From here, due to the compact embedding of \( H^1(S^1) \) into \( L^2(S^1) \), there exists a constant \( B \) such that
\[
u^{(k+m)/2}(x, t_n) \to B \text{ a.e. in } S^1 \quad \text{and} \quad \int_{S^1} u^{(k+m)/2}(x, t_n) \, dx \to B. \quad (2)
\]

Consequently from (1), we deduce that the sequence \( (u^{(k+m)/2}(\cdot, t_n)) \) is bounded in \( L^\infty(S^1) \) and thus, also the sequence \( (u(\cdot, t_n)) \).

Now, taking into account the uniform bound of \( u(\cdot, t_n) \) and that from (2), we infer
\[ u(x, t_n) \to B^{2/(k+m)} \text{ a.e. in } S^1, \]
and we deduce by Lebesgue’s theorem that
\[ \bar{u} = \int_{S^1} u(x, t_n) \, dx \to B^{2/(k+m)}. \]

and thus, \( B = \bar{u}^{(k+m)/2} \). Consequently, \( u(\cdot, t_n) - \bar{u} \to 0 \) a.e. in \( S^1 \) with the sequence \( u(\cdot, t_n) \) uniformly bounded in \( L^\infty(S^1) \). From now on, the proof follows as in the published paper. This argument was also used in (A. Jüngel, and I. Violet, First-order entropies for the Derrida-Lebowitz-Speer-Spohn equation, Discrete Contin. Dyn. Syst. B 8 (2007), 861-877) and we thank I. Violet for pointing out to us this gap.