

Nevanlinna class of entire functions in indeterminate Hamburger moment problems and in polynomial approximation on the real line

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Each entire function which can be an element of Nevanlinna's matrix for the indeterminate Hamburger moment problem is called a Nevanlinna's entire function. Such functions form the Nevanlinna class of entire functions $\mathcal{N}_0(\mathbb{R})$. We establish that $f' \in \mathcal{N}_0(\mathbb{R})$ for any $f \in \mathcal{N}_0(\mathbb{R})$ and that

$$\mathcal{N}_0(\mathbb{R}) = \left\{ \sum_{k=0}^{\infty} P_k^\mu(a) P_k^\mu(z) \mid a \in \mathbb{R}, \mu \in \text{indet } \mathcal{H} \right\},$$

where $\text{indet } \mathcal{H}$ is the family of all indeterminate measures on \mathbb{R} and $\{P_k^\mu\}_{k=1}^{\infty}$ is the sequence of orthonormal algebraic polynomials in $L_2(\mathbb{R}, d\mu)$. It is derived that an entire function in the de Branges solution (1959) of the Bernstein approximation problem (1924) can be chosen from the class $\mathcal{N}_0(\mathbb{R})$.

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