# Fast Algorithms for Boundary Integral Equations

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**Instructor:** Dr. Paolo Gatto
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**Office Hours:** right after class, or by appointment (please contact me by email.)

**Prerequisites:** This is a graduate level class for Master students of CES and Technical Mathematics. The intended audience should be familiar with basic concepts of functional analysis, Lebesgue integration, and numerical linear algebra.

**Course Description:** The main objective of this class is twofold: on one hand, to establish a connection between boundary value problems and boundary integral equations; on the other, to study discretization techniques for boundary integral operators. We shall begin by covering some classical results of potential theory (single and double layer potential, jump conditions) and see how they can transform a boundary value problem into an integral equation; we will restrict our attention to the Laplace and Helmholtz problems. We will discuss numerical discretization techniques, and focus on fast multipole methods; finally, we will see how those methods have led to the concept of hierarchical matrices.

**Workload:** 3.0 hours VU | 4.5 ECTS

**Textbook:** Since this is an advanced class at the forefront of research, the notes by the instructor found in TISS are intended as the main course material; additional reading material can also be found in TISS, and might be posted as the semester progresses. The following is an incomplete bibliography:

- R. Kress, *Linear Integral Equations*, Springer 2014;
- D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, Springer 2013;
- G.H. Golub and C.F. Van Loan, *Matrix Computations*, The John Hopkins University Press, 1996;
- G. Strang, *Computational Science and Engineering*, Wellesley Cambridge Press, 2012;
- L.N. Trefethen, *Approximation Theory and Approximation Practice*, SIAM 2013;
- P.G. Martinsson, *Fast Direct Solvers for Elliptic PDEs*, SIAM 2019.

**Assignments:** Exercises from the notes, that complement the material, will be assigned during the lectures. Students are expected to discuss their solutions in class, during exercise sessions. Although the assignments are neither collected nor graded, the assigned exercises constitute the bulk of the final exam.

**Attendance:** It is in students best interest to attend lectures; if you are unable to attend, you are personally responsible for the material covered in class.

**Grading/Exam:** To be decided, possibly dependent upon the size of class.

**Miscellanea:** This class is partially based on a workshop given by P.G. Martinsson at Dartmouth College in 2014, see `https://amath.colorado.edu/faculty/martinss/2014_CBMS/`. For lectures about approximation of functions by L.N. Trefethen see `https://people.maths.ox.ac.uk/trefethen/atapvideos.html`.

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