ACCURATE ERROR BOUNDS AND EQUIVALENCE OF SEMINORMS

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ABSTRACT

Error bounds for numerical approximations may be viewed as a comparison of two seminorms. An accurate bound then corresponds to the equivalence of the seminorms.

To exemplify this viewpoint, consider

$$(*) \qquad \sup_{\Omega} |u - I_h u| \leq C h^2 \sup_{\Omega} |u''|,$$

where Ω is a bounded interval, h indicates the length of intervals in a partition of Ω, and I_h denotes the corresponding Lagrange interpolation of first order. The maps

$$u \mapsto \sup_{\Omega} |u - I_h u| \quad \text{and} \quad u \mapsto h^2 \sup_{\Omega} |u''|$$

are seminorms on $C^0(\bar{\Omega})$ and $C^2(\bar{\Omega})$, respectively. Approximating in the max-norm a function $u \in C^0(\bar{\Omega}) \backslash C^2(\bar{\Omega})$ with functions form $C^2(\bar{\Omega})$, we see that the two seminorms are not equivalent. Therefore the bound (**??**) is not accurate in the aforementioned sense for a fixed h.

The talk will present and survey several accurate bounds in the error analysis of finite element methods.

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