PARALLEL BOUNDARY ELEMENT METHODS IN SPACE AND TIME ACCELERATED ON GPUS

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ABSTRACT

In modelling of many engineering problems solution to partial differential equations in unbounded domains is required. This is the case of heat equation, acoustics or Maxwell's equations of electromagnetism to name a few. Provided the fundamental solution (response to point sources) is available (in a closed form) one can reduce the partial differential equation from the unbounded domain to an integral equation along the boundary. The resulting operator can be understood as a continuous counterpart of the Schur complement of the related finite element method (FEM). Unlike in FEM, the correct behaviour of the solution at infinity is guaranteed without an additional cost. In case the equations are nonlinear in some bounded subdomains, BEM is coupled to FEM. Note that BEM is also advantageous in shape optimization problems [1].

In my opinion there are two reasons why BEM is much less popular than FEM. First of all, to evaluate the system matrix and the right-hand side vector one has to deal with singular integrals. Secondly, the system matrix is dense, hence a sparsification and/or parallel processing is necessary. We shall address both issues. Concerning the latter we shall present our methodology of parallel implementation of hierarchical matrices arising from a BEM sparsification. The overall computational time scales up to millions of 3d boundary DOFs and hundreds of CPU cores for stationary problems [2]. The approach is further extended to transient problems discretized simultaneously in space and time [3]. Unfortunately, the methods suffer from a suboptimal memory scalability. To avoid the latter and, at the same time, address the issue of singular integrals evaluation, we present a matrix-free variant of the conventional BEM where the intensive numerical quadratures within both the dense system matrix and an operator preconditioner [4] are implemented on GPUs. I believe this is an upcoming direction in the BEM software development.

References

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