REDUCED, ALL-AT-ONCE, AND VARIATIONAL FORMULATIONS OF INVERSE PROBLEMS AND THEIR ITERATIVE SOLUTION

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ABSTRACT

The conventional way of formulating inverse problems such as identification of a (possibly infinite dimensional) parameter, is via some forward operator, which is the concatenation of the observation operator with the parameter-to-state-map for the underlying model.

Recently, all-at-once formulations have been considered as an alternative to this reduced formulation, avoiding the use of a parameter-to-state map, which would sometimes lead to too restrictive conditions. Here the model and the observation are considered simultaneously as one large system with the state and the parameter as unknowns [1].

A still more general formulation of inverse problems, containing both the reduced and the all-at-once formulation, but also the well-known and highly versatile so-called variational approach (not to be mistaken with variational regularization) as special cases, is to formulate the inverse problem as a minimization problem (instead of an equation) for the state and parameter. Regularization can be incorporated via imposing constraints and/or adding regularization terms to the objective [2]. In this talk we will provide some new application examples of minimization based formulations in the context of identifying a diffusion parameter in an elliptic PDE; depending on which type of observations we have, we end up with (a) a model problem in inverse groundwater filtration GWF (b) impedance acoustic tomography IAT a novel hybrid imaging method, (c) the classical inverse problem of electrical impedance tomography, also known as Calderón's problem.

We will consider iterative regularization methods resulting from the application of gradient or Newton type iterations to such minimization based formulations and provide convergence results. In doing so, instead of regularizing the minimization problem and then applying standard iterative optimization methods, we regularize *by* iterating, more precisely by early stopping.

This is joint work with Kha Van Huynh, University of Klagenfurt.

References

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