MULTILEVEL DECOMPOSITIONS IN NEGATIVE ORDER SOBOLEV SPACES AND APPLICATIONS

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ABSTRACT

In this talk we present novel results [1] on multilevel decompositions of piecewise polynomial spaces in negative order Sobolev spaces. We discuss the basic ideas of the stability proof for sequences of uniform simplicial meshes and sequences of adaptively generated meshes. The latter requires the introduction of a local projection operator in negative order spaces.

As a first application we consider the definition of local multilevel diagonal preconditioners which can be evaluated in linear computational complexity, thus, leading to, e.g., an efficient solver for (adaptive) boundary elements for the screen problem.

As a second application we review the construction of equivalent multilevel norms that allow to efficiently evaluate negative order Sobolev norms of piecewise polynomial functions. The definition of such multilevel norms requires a local quasi-interpolation operator similar to the projection operator mentioned above. The novel multilevel norms can be used for problems that involve residual minimization in negative order spaces.

Finally, we show that the projection resp. quasi-interpolation operators can be used to regularize minimum residual methods such as least-squares finite element methods or discontinuous Petrov–Galerkin methods with optimal test functions [2]. This allows to use non-square integrable loads as well as point sources.

References

[1] T. Führer. *Multilevel decompositions and norms for negative order Sobolev spaces*, Math. Comp. 91 (2021), 183–218.

[2] T. Führer, N. Heuer, M. Karkulik. *MINRES for second-order PDEs with singular data*, SIAM J. Numer. Anal. 60 (2022), 1111–1135.

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