LOWER EIGENVALUE BOUNDS FOR THE HARMONIC AND BI-HARMONIC OPERATOR

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ABSTRACT

Recent advances in the nonconforming FEM approximation of elliptic PDE eigenvalue problems include the guaranteed lower eigenvalue bounds (GLB) and its adaptive finite element computation. Like guaranteed upper eigenvalue bounds with conforming finite element methods, GLB arise naturally from the min-max principle, also named after Courant, Fischer, Weyl.

The first part introduces the derivation of GLB for the simplest second-order and fourth-order eigenvalue problems with relevant applications, e.g., for the localization of in the critical load in the buckling analysis of the Kirchhoff plates. The second part studies an optimal adaptive mesh-refining algorithm for the effective eigenvalue computation for the Laplace and bi-Laplace operator with optimal convergence rates in terms of the number of degrees of freedom relative to the concept of nonlinear approximation classes. The third part presents a modified hybrid high-order (HHO) eigensolver in the spirit of Carstensen, Ern, and Puttkammer [Numer. Math. 149, 2021] that directly computes guaranteed lower eigenvalue bounds under the idealized hypothesis of exact solve of the generalized algebraic eigenvalue problem and a mild explicit condition on the maximal mesh-size in a simplicial mesh.

The error analysis allows for a priori quasi-best approximation and L^2 error estimates as well as a stabilization-free reliable and efficient a posteriori error control. The associated adaptive mesh-refining algorithm performs well in computer benchmarks with striking numerical evidence for optimal higher convergence rates.

The topics reflect joint work with Sophie Puttkammer (Berlin), Ngoc Tien Tran (Jena), and Benedikt Gräßle (Berlin).

References

[1] C. Carstensen, A. Ern, and S. Puttkammer, *Guaranteed lower bounds on eigenvalues of elliptic operators with a hybrid high-order method*, Numer. Math. 149 (2021), 273–304.

[2] C. Carstensen and D. Gallistl, *Guaranteed lower eigenvalue bounds for the biharmonic equation*, Numer. Math. 126 (2014), 33–51.
[3] C. Carstensen and J. Gedicke, *Guaranteed lower bounds for eigenvalues*, Math. Comp. 83 (2014), 2605–2629.
[4] C. Carstensen, B. Gräßle, and N. T. Tran, *Adaptive hybrid high-order method for guaranteed lower eigenvalue bounds*. work in progress, 2022.
[5] C. Carstensen and S. Puttkammer, *Direct guaranteed lower eigenvalue bounds with optimal a priori convergence rates for the bi-Laplacian*, 2021. preprint (arXiv:2105.01505v2).
[6] C. Carstensen and S.Puttkammer, *Adaptive guaranteed lower eigenvalue bounds with optimal convergence rates*, (2022). preprint (arXiv:2203.01028).
[7] C. Carstensen, Q. Zhai, and R. Zhang, *A skeletal finite element method can compute lower eigenvalue bounds*, SIAM J. Numer. Anal. 58 (2020), 109–124.

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