

SPACE-TIME FINITE ELEMENT METHODS FOR PARABOLIC OPTIMAL CONTROL PROBLEMS

ULRICH LANGER*, ANDREAS SCHAFELNER

ABSTRACT

We present, analyze and test locally stabilized space-time finite element (fe) methods on fully unstructured simplicial space-time meshes for the numerical solution of space-time tracking parabolic optimal control problems with the standard L_2 -regularization. We derive a priori discretization error estimates in terms of the local mesh-sizes for shape-regular meshes. The adaptive version is driven by local residual error indicators, or by local error indicators derived from a new functional a posteriori error estimator. The latter provides a guaranteed upper bound of the error, but is more costly than the residual error indicators. We perform numerical tests for benchmark examples having different features. In particular, we consider a discontinuous target in form of a first expanding and then contracting ball in 3d that is fixed in the 4d space-time cylinder. These results will partly be published in [2]. We also refer to [1] and [3, 4] for related results for the state equation and different optimal control problems, respectively. The latter results are based on discrete inf-sup conditions, whereas the results of this talk rely on the ellipticity of the discrete bilinear form that is related to the up- and downwind fe discretization of reduced optimality system.

REFERENCES

- [1] U. Langer, A. Schafelner. *Adaptive space-time finite element methods for non-autonomous parabolic problems with distributional sources*, Comput. Methods Appl. Math., 20 (2020), 677–69
- [2] U. Langer, A. Schafelner. *Adaptive space-time finite element methods for parabolic optimal control problems*, J. Numer. Math. (2022), to appear.
- [3] U. Langer, O. Steinbach, F. Tröltzsch, H. Yang. *Unstructured space-time finite element methods for optimal control of parabolic equation*, SIAM J. Sci. Comp. 43 (2021), A744–A771.
- [4] U. Langer, O. Steinbach, F. Tröltzsch, H. Yang. *Space-time finite element discretization of parabolic optimal control problems with energy regularization*, SIAM J. Numer. Anal. 59 (2021), 675–695.

* JOHANNES KEPLER UNIVERSITY LINZ, ULRICH.LANGER@JKU.AT