AN OPTIMAL GOAL-ORIENTED ADAPTIVE FEM WITH QUADRATIC GOAL FUNCTIONAL

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ABSTRACT

On a bounded Lipshitz domain Ω, we consider a linear elliptic PDE

$$\begin{aligned} -\mathrm{div}(A\nabla u) + \boldsymbol{b}\cdot\nabla u + cu &= f + \mathrm{div}(\boldsymbol{f}) \qquad \text{in } \Omega,\\ u &= 0 \qquad\qquad\qquad \text{on } \partial\Omega, \end{aligned}$$

with solution $u \in H^1_0(\Omega)$ and a quadratic goal functional of the form

$$G(u) := \langle \mathcal{K}u, u\rangle_{H^{-1}\times H^1_0},$$

where $\mathcal{K}\colon H^1_0(\Omega) \to H^{-1}(\Omega)$ is linear and bounded.

We propose a goal-oriented adaptive FEM algorithm (GOAFEM) that approximates the quantity of interest $G(u)$ by values $G(u_\ell)$ in the ℓ-th step. To this end, our GOAFEM solves the primal as well as a linearized dual problem and employs a variant of the marking strategy from [1] for linear goal functionals. For bounded $\mathcal{K}$, we show that the error in the goal functional converges, i.e.,

$$|G(u) - G(u_\ell)| \le C\,\eta_\ell(u_\ell)\big[\eta_\ell(u_\ell)^2 + \zeta_\ell(z_\ell)^2\big]^{1/2} \longrightarrow 0 \quad \text{as } \ell \to \infty,$$

where the *a posteriori* error estimators $\eta_\ell(u_\ell)$ and $\zeta_\ell(z_\ell)$ bound the discretization error of the primal and dual problem, respectively. Moreover, if $\mathcal{K}$ is compact, the goal error even converges linearly with optimal algebraic rate $\alpha > 0$, i.e.,

$$\eta_\ell(u_\ell)\big[\eta_\ell(u_\ell)^2 + \zeta_\ell(z_\ell)^2\big]^{1/2} \le C\,(\#\mathcal{T}_\ell - \#\mathcal{T}_0)^{-\alpha}.$$

Numerical examples underline our theoretical findings.

References

[1] Feischl, M. and Praetorius, D. and van der Zee, K. G. *An abstract analysis of optimal goal-oriented adaptivity*, SIAM J. Numer. Anal. 54 (2016), 1423–1448.

[2] Becker, R. and Innerberger, M. and Praetorius, D. *Optimal convergence rates for goal-oriented FEM with quadratic goal functional*, Comput. Methods Appl. Math. 21 (2020), 267–288.

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