A SEMISMOOTH NEWTON APPROACH TO MODEL VISCOPLASTIC LAVA FLOW USING THE DISCONTINUOUS GALERKIN METHOD

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ABSTRACT

Our goal is to simulate lava flow to obtain hazard maps that help to mitigate their associated risks. To do this, we model lava flow using the shallow-water equations for a viscoplastic material. We use the Bingham model in which the material behaves as a solid or fluid depending if the stresses are below or above a certain threshold. Although there have been many studies of this problem, most of these works use explicit methods with simplified empirical models. The main difficulty of this problem is the coupling of the shallow-water equations with the viscoplastic constitutive laws and the high computational effort needed in its solution.

In our work to accommodate non-uniform grids and complicated geometries, we use the discontinuous Galerkin method to solve this problem. This method is also attractive due to its high-order accuracy, high parallelization, and the ability to capture the discontinuities of the exact solution. Additionally, we treat the discontinuities in the interfaces between elements with numerical fluxes that ensure a stable solution of nonlinear hyperbolic equations. To couple the Bingham model with the shallow-water equations, we regularize the problem [1]. This regularization yields a semi-smooth system of Newton differentiable equations where we expect local superlinear convergence in each time iteration solution. Numerical examples are performed for the usual benchmarks of the shallow-water equations to show the effectiveness of our approach.

References

[1] J. De Los Reyes, S. González. *A combined BDF-semismooth Newton approach for time-dependent Bingham flow*, Numerical Methods for Partial Differential Equations, 28 3 (2012), 834–860

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