A STRUCTURE PRESERVING LOW-REGULARITY INTEGRATOR FOR THE KORTEWEG–DE VRIES EQUATION

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ABSTRACT

Most classical constructions of time-stepping methods for evolution equations are based on Taylor series expansions. As a result their convergence is dependent on large amount of differentiability (regularity) in the solution, and genuine low-regularity phenomena cannot be resolved reliably. This has led to the recent development of bespoke resonance-based methods which can achieve guaranteed convergence under more lenient requirements on the regularity of initial data. Amongst the current state-of-the-art in low-regularity schemes for the KdV equation is recent work by Hofmanová & Schratz [2], whose method achieves first-order convergence in H^1 with initial data in H^3.

In this talk, we will consider numerical methods for the initial value problem of the Korteweg–De Vries (KdV) equation in the low-regularity regime. Complete integrability means that the KdV equation has an infinite number of first integrals. In the context of geometric numerical integration [1], we will present and analyse a novel symplectic resonance-based integrator for the KdV equation which can preserve the first two integrals (momentum and energy) over long times. Additionally, we will show that this construction leads to improved convergence properties over current state-of-the-art schemes even for H^3 initial data. The advantageous properties of this new method will be demonstrated both from a theoretical point of view and in computational experiments.

References

[1] E. Hairer, C. Lubich, G. Wanner. *Geometric Numerical Integration: Structure-Preserving Algorithms for Ordinary Differential Equations*, Springer, 2013.

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