NONLINEAR MODEL ORDER REDUCTION FOR PARAMETRIZED HYPERBOLIC CONSERVATION LAWS IN A SPACE-TIME DOMAIN

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ABSTRACT

Due to moving fronts or shock interactions, model order reduction for parametrized hyperbolic conservation laws still constitutes a severe difficulty. Methods solely based on projection onto linear subspaces of the solution space suffer from a slow decay of the Kolmogorov N-width of the solution manifold for hyperbolic problems, as for instance shown in [3] for the wave equation.

Therefore, we propose an approach based on a nonlinear transformation of the underlying space-time domain to transform and align discontinuities and shocks. By considering space-time solutions, we circumvent the problem of shock interactions, which are directly incorporated in the solution snapshots. The diffeomorphisms transforming the space-time domain are parametrized via vector fields whose time-evolution is determined by a geodesic flow in the diffeomorphism group, see [1]. Similar to the approach described in [2], methods from linear model order reduction are applied to the Lie algebra of vector fields to obtain a reduced order model.

References

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