A GALERKIN METHOD BASED ON COHERANT STATE FOR HIGH-FREQUENCY HELMHOLTZ PROBLEMS

T. CHAUMONT-FRELET*, V. DOLEAN, M. INGREMEAU

ABSTRACT

For a given frequency $k > 0$, the Helmholtz equation

$$\begin{cases} -\mu k^2 u - \boldsymbol{\nabla} \cdot (\boldsymbol{A} \boldsymbol{\nabla} u) & = & f & \text{in } \mathbb{R}^d, \\ \dfrac{1}{|\boldsymbol{x}|} \boldsymbol{\nabla} u \cdot \boldsymbol{x} - iku & \to & 0 & \text{as } |\boldsymbol{x}| \to \infty, \end{cases}$$

models the propagation of a time-harmonic wave. u is the uknown, f is a given right-hand side, and μ and $\boldsymbol{A}$ are smooth coefficients that are respectively equal to 1 and $\boldsymbol{I}$ outside of a ball of radius $\ell > 0$.

While the Helmholtz equation is of great interest due to many applications, it is still very challenging to solve when the frequency is large. In particular, finite element methods require $(k\ell)^d$ degrees of freedom to accurately approximate the solution, assuming the polynomial degree is "large enough". Boundary integral equations can be employed to obtain an accurate representation with $(k\ell)^{d-1}$ degrees of freedom, but only in the case where the coefficients μ and $\boldsymbol{A}$ are (piecewise) constant.

In this presentation, I will describe a novel Galerkin method that employs "coherant states" as basis functions (as opposed to polynomials in finite element methods). These coherent states are of the form

$$\Psi_{k,\boldsymbol{x}_0,\boldsymbol{\xi}_0}(\boldsymbol{x}) := (k\pi)^{-d/4} e^{-\frac{k}{2}|\boldsymbol{x}-\boldsymbol{x}_0|^2} e^{ik\boldsymbol{\xi}_0\cdot(\boldsymbol{x}-\boldsymbol{x}_0)}$$

where $\boldsymbol{x}_0, \boldsymbol{\xi}_0 \in \mathbb{R}^d$ are carefully chosen parameters. I will show that for a large set of right-hand sides f (including those resulting from the scattering of a plane wave), the solution can be approximated with only $(k\ell)^{d-1/2}$ degrees of freedom using coherent states, which is the key asset of the proposed method. I will also discuss the challenges associated with the use of coherent states and present one-dimensional numerical experiments that highlight the potential of the method.

* Inria Université Côte d'Azur, LJAD, CNRS, theophile.chaumont@inria.fr