A SPACE-ANGLE DGFEM FOR SOLVING THE LINEAR BOLTZMANN EQUATION IN THE PRESENCE OF A MAGNETIC FIELD USING AN ANGULAR MULTIGRID PRECONDITIONER

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ABSTRACT

We present an efficient solution method for charged particle transport in a magnetic field, described by the Linear Boltzmann Equation (LBE) with an additional force term: [1].

$$\begin{aligned} &\hat{\Omega}\cdot\vec{\nabla}\phi(\vec{r},E,\hat{\Omega})+\sigma_t(\vec{r},E)\phi(\vec{r},E,\hat{\Omega})-\frac{\partial S\phi(\vec{r},E,\hat{\Omega})}{\partial E}\\ &\frac{q}{p(E)}\vec{\nabla}_s\cdot(\vec{v}_s\phi(\vec{r},E,\hat{\Omega}))=Q_{scat}(\vec{r},E,\hat{\Omega})+Q_{ex}(\vec{r},E,\hat{\Omega}), \end{aligned} \tag{1}$$

where $\vec{\nabla}_s\cdot$ denotes the divergence over the surface of the unit sphere, $p(E)$ is the relativistic particle momentum and $\vec{v}_s$ is defined as

$$\vec{v}_s=-\frac{(\vec{\Omega}\times\vec{B})_z}{\sin\theta}\hat{\theta}+\frac{\left(\vec{\Omega}\times(\vec{\Omega}\times\vec{B})\right)_z}{\sin\theta}\hat{\chi}. \tag{2}$$

In our novel approach the dgFEM is used to discretize the LBE in space, energy, and angle. Standard polynomials are used in space and energy. Projections of linear basis functions on the faces of an octahedron onto the sphere are used as expansion an angle. The sphere is hierarchically refined to the level required. A space-angle sweep is used as smoother within an angular multigrid scheme. This scheme is used as a preconditioner for a Krylov method. To test the efficiency, the method of manufactured solutions was applied to various test cases with a range of material properties and magnetic field strengths. Results show that our method is much more effective than regular source iteration based methods.

References

[1] J. St. Aubin, et al. Medical physics 424.2 (2015), pp. 780-793.

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