RIGOROUS EIGENVALUE ESTIMATION FOR THE STOKES DIFFERENTIAL OPERATORS

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ABSTRACT

The problem of solution verification for nonlinear partial differential equations, e.g., the Navier-Stokes equation, urges the need of rigorous eigenvalue estimation for differential operators. As the main topic of this talk, a general framework proposed in [1, 2, 3] for rigorous eigenvalue estimation will be introduced. Such a framework has been successfully applied to the eigenvalue problem of the Laplace operator, the Biharmonic operator, the Stokes operator, the Steklov operator. For the Stokes operator, both conforming and non-conforming finite element methods are utilized to obtain explicit eigenvalue bounds [4, 5]. Finally, as an application of the rigorous eigenvalue estimation for the Stokes operator, there is a case report for the solution verification of the stationary Navier-Stokes equation over a 3D non-convex domain [6].

References

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