OPTIMAL CONTROL OF IRREGULAR DRIFT IN A FOKKER-PLANCK EQUATION

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ABSTRACT

In this talk we consider a bilinear optimal control problem subject to the Fokker-Planck equation

$$y'(t) + \operatorname{div}(y(t)u(t)\alpha) - \operatorname{div}(\sigma(t)\sigma(t)^\top \nabla y(t)) = 0, \qquad y(0) = y_0.$$

The control u acts as the temporal amplitude for the spatial drift direction α. The defining feature of this problem is that we only assume $\alpha \in \mathrm{BV}$ in order to accommodate W-shaped potentials; also, σ is not assumed to be bounded, and we aim to not need $\sigma\sigma^\top$ uniformly positive definite. The state equation is well posed with weak solutions via the technique of renormalized solutions [1]. However, due to the combination of bilinear control and low regularity of α, the sensitivity equation is not directly tractable by the same methods. In fact, we obtain only the nonstandard notion of renormalized, but not weak solutions for the linearized and adjoint equations. We show that this concept still permits to obtain continuity properties of the control-to-state operator and optimality conditions for the optimal control problems, and how so.

References

[1] D.J. Luo. *Fokker-Planck type equations with Sobolev diffusion coefficients and BV drift coefficients*, Acta. Math. Sin.-English Ser. 29 (2013) 303–314. https://doi.org/10.1007/s10114-012-1302-x

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