A POSTERIORI ERROR CONTROL IN THE MAXIMUM NORM FOR CONVECTION-DOMINATED CONVECTION-DIFFUSION EQUATIONS

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ABSTRACT

Standard and stabilized finite element approximations are considered on shape-regular meshes for singularly perturbed convection-diffusion equations. Our initial result is that natural maximum-norm residual estimators of type [1] reliably control the error in the maximum norm, assuming suitable estimates of the Green's function hold, such as in [2]. On the other hand, residual-type estimators in the energy norm are only efficient up to a dual norm of the convective error [3]. A main contribution of the paper is to analogously define a suitable dual seminorm of the convective error. Having defined such a dual norm, we then define the total error as the originally targeted maximum norm of the error plus the dual seminorm of the convective error plus standard data oscillation terms. Our a posteriori error estimator is then shown to be equivalent to the total error (up to a logarithmic factor). Numerical experiments illustrate the behavior and performance of our estimators in the context of uniform and adaptive mesh refinement. In particular, they show that the estimators may vastly overestimate the error in the maximum norm alone, but they closely track the total error as predicted by our theory. Adaptive refinement based on our error indicators is also shown to do an effective job at automatically resolving standard model problems whose solutions include strong layers.

References

[1] A. Demlow, N. Kopteva. *Maximum-norm a posteriori error estimates for singularly perturbed elliptic reaction-diffusion problems*, Numer. Math. 133 (2016), 707–742.

[2] S. Franz, N. Kopteva. *Green's function estimates for a singularly perturbed convection-diffusion problem*, J. Differential Equations 252 (2012), 1521–1545.

[3] L. Tobiska, R. Verfürth, *Robust a posteriori error estimates for stabilized finite element methods*, IMA J. Numer. Anal. 35 (2015) 1652–1671.

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