DIVERGENCE-FREE FINITE ELEMENT METHODS FOR AN INVISCID FLUID MODEL

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ABSTRACT

In this talk I will review some recent results ([1, 2]) on the stabilisation of linearised incompressible invisid flows (or, with a very small viscosity). The common point of the two approaches is the aim of proving the following pressure-robust estimate:

$$\|\boldsymbol{u} - \boldsymbol{u}_h\|_{L^2} \leq C h^{k+\frac{1}{2}} |\boldsymbol{u}|_{H^{k+1}} , \tag{1}$$

where $\boldsymbol{u}$ is the exact velocity and $\boldsymbol{u}_h$ is its finite element approximation. This estimate mimicks what has been achieved for stabilised methods for the convection-diffusion equation in the past. Nevertheless, up to the best of our knowledge, this is the first time this type of estimate is obtained in a pressure-robust way. Both $H(div)$ and H^1-conforming finite element spaces are considered. While for the former an upwind method provides sufficient control on the convective derivative to prove (1), in the latter case the need for a stabilisation of the vorticity appears. Numerical results will be presented and the present approach will be compared to the classical residual-based SUPG stabilisation. If time allows, I will finish the talk with more recent developments on pressure-robust CIP stabilisation for this kind of problem.

This work is a collaboration with N. Ahmed (Gulf University for Science and Technology, Kuwait), E. Burman (UCL, UK), J. Guzmán (Brown, USA), and A. Linke and C. Merdon, (WIAS, Berlin).

References

[1] G.R. Barrenechea, E. Burman, and J. Guzmán, *Well-posedness and $H(div)$-conforming finite element approximation of a linearised model for inviscid incompressible flow*, *Math. Mod. Meth. Appl. S.*, **30**(5), pp. 847–865, 2020.

[2] N. Ahmed, G.R. Barrenechea, E. Burman, J. Guzmán, A. Linke, and C. Merdon, *A pressure-robust discretization of Oseen's equation using stabilization in the vorticity equation*, *SIAM Journal on Numerical Analysis*, **59** (5), pp. 2746–2774, 2021.

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